Math 222:09 Spring 2010

Working with the infinite series: $\sum_{n=1}^{\infty} a_n$

This is a guide for how to evaluate if an infinite series is convergent or divergent.

1^{st} Check the n^{th} term of $\sum a_n$. If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges, and you are done (Use T.F.D.). However if $\lim_{n\to\infty} a_n = 0$, then proceed to step 2.

2^{nd} Check to see if the series is harmonic, geometric, or a p-series.

- Geometric Series: it will look like $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^n$. Now we check if $|r| < 1$.
- p-series/Harmonic: it will look like $\sum_{n=1}^{\infty} \frac{1}{n^p}$. Now we check to see if $p > 1$.

If it is none of these, go to step 3, step 6, or step 8.

If the series is all positive numbers (after some point):

3^{rd} Try comparing the series to one that you know. Usually the series you want to compare to are geometric series, p-series, and/or the series made by only using the dominating terms from your numerator and denominator. *Make sure the series you're comparing to is one you can evaluate.*
Use either the Comparison Test or Limit Comparison Test.

4^{th} If that didn't work, try either the integral test, ratio test, or root test.
- Try the ratio test when there are terms like $n!$ or c^n.
- Try the root test when there are terms like n^n, or even a function $f(n)^{5n}$.
- Try the integral test when the a_n can be written as some easily integrable $f(n)$. (Integral test works well with logs and ln.)

5^{th} If nothing has worked so far consider:
- More creative comparison.
- Can you split the series up along addition/subtraction into 2 *convergent* series?
- Using partial sums (maybe they will telescope).

If the series alternates between positive and negative terms (after some point):

6^{th} Try the alternating series test. If that doesn't work,

7^{th} Write $\sum_{n=1}^{\infty} |a_n|$ and then go to step 3. (You're hoping for absolute convergence here.)

8^{th} If the series is *not* eventually alternating and *not* eventually all positive, your first choices are either to hope for absolute convergence or to work with the partial sums.