

NAME (Printed): _____

Math 304-2 Linear Algebra Fall 2024 Quiz 1 Feingold

INSTRUCTIONS: Show all necessary work for each problem.

$$\text{Let } A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 3 & 5 \\ 2 & 3 & -1 & 0 \end{bmatrix}.$$

- (1) (5 Points) Use row reduction to **Reduced Row Echelon Form** (RREF) to find all solutions $X \in \mathbf{F}^4$ to the linear system $AX = 0_1^4$ in terms of some **free variables**.

(2) (5 Points) Let $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \in \mathbf{F}^4$. Use row reduction to Reduced Row Echelon Form (RREF) to find the conditions on the entries of B required for the linear system $AX = B$ to be **consistent**.

1. (5 Points) Use row reduction to **Reduced Row Echelon Form** (RREF) to solve the linear system by row reducing

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ -1 & 1 & 3 & 5 & 0 \\ 2 & 3 & -1 & 0 & 0 \end{array} \right] \text{ to } \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \text{ so } \begin{cases} x_1 = 2r + 3s \\ x_2 = -r - 2s \\ x_3 = r \in \mathbf{F} \\ x_4 = s \in \mathbf{F} \end{cases}.$$

The solution set in terms of **free variables** is $\left\{ X = \begin{bmatrix} 2r + 3s \\ -r - 2s \\ r \\ s \end{bmatrix} \in \mathbf{F}^4 \mid r, s \in \mathbf{F} \right\}.$

2. (5 Points) For $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \in \mathbf{F}^4$ find conditions on the entries of B for the system

$AX = B$ to be **consistent**. Row reduce the augmented matrix and find consistency conditions from rows with all zeros on the left side:

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & b_1 \\ 1 & 2 & 0 & 1 & b_2 \\ -1 & 1 & 3 & 5 & b_3 \\ 2 & 3 & -1 & 0 & b_4 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 2b_1 - b_2 \\ 0 & 1 & 1 & 2 & -b_1 + b_2 \\ 0 & 0 & 0 & 0 & 3b_1 - 2b_2 + b_3 \\ 0 & 0 & 0 & 0 & b_1 + b_2 - b_4 \end{array} \right] \begin{array}{l} \text{is consistent iff} \\ 0 = 3b_1 - 2b_2 + b_3 \\ \text{and} \\ 0 = b_1 + b_2 - b_4 \end{array}.$$
