

NAME (Printed): _____

Math 304-2 Linear Algebra Fall 2024 Quiz 4 Feingold

INSTRUCTIONS: Show all calculations needed to justify your answers.

- (1) (5 Pts) Let m, n, p be given positive integers and let $A \in \mathbb{F}_n^m$ be a given $m \times n$ matrix fixed for this problem. Then A determines a set of $n \times p$ matrices $W = \{B \in \mathbb{F}_p^n \mid AB = 0_p^m\}$. Show W is a subspace of \mathbb{F}_p^n .

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- (2) (5 Pts) Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \in \mathbb{F}_2^2$. Find $W = \left\{ B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{F}_2^2 \mid AB = 0_2^2 \right\}$ by solving a linear system.

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- (1) (5 Pts) Let m, n, p be given positive integers and let $A \in \mathbb{F}_n^m$ be a given $m \times n$ matrix fixed for this problem. Then A determines a set of $n \times p$ matrices

$$W = \{B \in \mathbb{F}_p^n \mid AB = 0_p^m\}.$$

Show W is a subspace of \mathbb{F}_p^n .

SOLUTION: First show W is closed under addition. Let $B_1, B_2 \in W$, so $AB_1 = 0_p^m$ and $AB_2 = 0_p^m$. Then $A(B_1 + B_2) = AB_1 + AB_2 = 0_p^m + 0_p^m = 0_p^m$ means $B_1 + B_2 \in W$.

Next show W is closed under scalar multiplication. Let $B \in W$ so $AB = 0_p^m$, and let $r \in \mathbb{F}$. Then $A(rB) = r(AB) = r0_p^m = 0_p^m$ means $rB \in W$.

Finally, $0_p^n \in W$ since $A0_p^n = 0_p^m$.

- (2) (5 Pts) Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \in \mathbb{F}_2^2$. Find $W = \left\{ B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{F}_2^2 \mid AB = 0_2^2 \right\}$ by solving a linear system.

SOLUTION: To find all $B \in \mathbb{F}_2^2$ such that $AB = 0_2^2$, write out the condition as

$$\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} (a + 3c) & (b + 3d) \\ (3a + 9c) & (3b + 9d) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

This gives us four equations in four variables which we solve by row reduction as follows:

$$\text{Row reduce } \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 3 & 0 & 9 & 0 & 0 \\ 0 & 3 & 0 & 9 & 0 \end{array} \right] \text{ to } \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \text{ so } \begin{array}{l} a = -3c \\ b = -3d \\ c \in \mathbb{F} \text{ free} \\ d \in \mathbb{F} \text{ free} \end{array}$$

$$\text{so } W = \left\{ B = \begin{bmatrix} -3c & -3d \\ c & d \end{bmatrix} \in \mathbb{F}_2^2 \mid c, d \in \mathbb{F} \right\}.$$