Mon. May 4, Math 304-6 Feingold /1

Final Exam Review:

$L: V \to W$ lin. map is <u>injective</u> when

$L(v_1) = L(v_2)$ implies $v_1 = v_2$.

Th. L is inj iff $\mathrm{Ker}(L) = \{0_V\}$

iff $\dim(\mathrm{Ker}(L)) = 0$.

L is <u>surjective</u> when $\mathrm{Range}(L) = W$

iff $\forall w \in W, \exists v \in V$ s.t. $L(v) = w$

iff $\dim(\mathrm{Range}(L)) = \dim(W)$.

$\dim(V) = \dim(\mathrm{Ker}(L)) + \dim(\mathrm{Range}(L))$

$\text{Ker}(L) = \{ v \in V \mid L(v) = \theta_W \} \leq V$ $\underline{/2}$

For $A \in \mathbb{R}^m_n$ have lin. map $L_A : \mathbb{R}^n \to \mathbb{R}^m$ defined by $L_A(X) = AX$.

$\text{Nul}(A) = \{ X \in \mathbb{R}^n \mid AX = 0^m_1 \} = \text{Ker}(L_A)$

$\text{Col}(A) = \langle \text{Col}_1(A), \ldots, \text{Col}_n(A) \rangle$

$= \left\{ \sum_{j=1}^{n} x_j \text{Col}_j(A) \;\middle|\; x_j \in \mathbb{R} \right\} = \{ AX \in \mathbb{R}^m \mid X \in \mathbb{R}^n \}$

$= \text{Range}(L_A)$.

$$\begin{vmatrix} -6 & -2 & 2 & 7 & -4 \\ -9 & 0 & -9 & -3 & 4 \\ -3 & 0 & 0 & -7 & 3 \\ x & -2 & 2 & 9 & 4 \\ 4 & 0 & 0 & 0 & -6 \end{vmatrix} = \begin{vmatrix} -2 & -2 & 2 & 7 & -10 \\ -1 & 0 & -9 & -3 & -8 \\ 1 & 0 & 0 & -7 & -3 \\ x & -2 & 2 & 9 & 4 \\ 4 & 0 & 0 & 0 & -6 \end{vmatrix} = \; 3$$

$$\begin{matrix} 8 & 0 & 0 & 0 & -12 \end{matrix} \qquad \begin{matrix} 2 & 0 & 0 & -14 & -6 \end{matrix}$$

$$\begin{vmatrix} 0 & -2 & 2 & -7 & -16 \\ 0 & 0 & -9 & -10 & -11 \\ 1 & 0 & 0 & -7 & -3 \\ x & -2 & 2 & 9 & 4 \\ 4 & 0 & 0 & 0 & -6 \end{vmatrix} = 4 \begin{vmatrix} -2 & 2 & -7 & -16 \\ 0 & -9 & -10 & -11 \\ 0 & 0 & -7 & -3 \\ -2 & 2 & 9 & 4 \end{vmatrix} - 6 \begin{vmatrix} 0 & -2 & 2 & -7 \\ 0 & 0 & -9 & -10 \\ 1 & 0 & 0 & -7 \\ x & -2 & 2 & 9 \end{vmatrix}$$

$$= \text{Const.} - 6(-x) \begin{vmatrix} -2 & 2 & -7 \\ 0 & -9 & -10 \\ 0 & 0 & -7 \end{vmatrix}$$

cofactor expand by row 5

$$f(x) = \text{Const} - 6(126)x$$

$$f'(x) = -756$$

Sabspace: Let $W = \{p(t) \in \mathbb{P}_3 \mid p'(1) = 1\}$ 4

Is W a sabspace of $\mathbb{P}_3$? No, $0 \notin W$.

Also, W is not closed under $+$ or $\cdot$.

Try $U = \{p(t) \in \mathbb{P}_3 \mid p'(1) = 0\}$.

$0 \in U$. If $p(t), q(t) \in U$ then

$(p+q)'(1) = p'(1) + q'(1) = 0 + 0 = 0$ so

$p+q \in U$. $\forall c \in \mathbb{R}$, $(cp)(t) \in U$ since

$(cp)'(1) = c(p'(t)) = c(0) = 0$. Yes $U \leq \mathbb{P}_3$

Let $W = \left\{ \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{=A} \in \mathbb{R}^2_2 \;\middle|\; \begin{array}{l} \det(A) = 0 \\ ad - bc = 0 \end{array} \right\}$ 5

<u>Note</u>: $O^2_2 \in W$ since $\det \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

If $A \in W$, $e \in \mathbb{R}$, $\det(eA) = \det \begin{bmatrix} ea & eb \\ ec & ed \end{bmatrix}$

$= e^2(ad - bc) = e^2(0) = 0$

But for $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \in W$

we have $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \notin W$ since $\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$.

So W is <u>not</u> a subspace of $\mathbb{R}^2_2$.

W is not closed under $+$.

For any $\theta \neq v \in \mathbb{R}^n$, $\|v\| = \sqrt{v \cdot v} > 0$

$\forall c \in \mathbb{R}$, $\|cv\| = \sqrt{(cv)\cdot(cv)} = \sqrt{c^2}\sqrt{v \cdot v} = |c|\,\|v\|$

If $v = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ then $v \cdot v = \sum_{i=1}^{n} a_i a_i = \sum_{i=1}^{n} a_i^2$

so $\|v\| = \sqrt{\sum_{i=1}^{n} a_i^2}$. For $c = \frac{1}{\|v\|}$

get $\|cv\| = 1$. so $\frac{v}{\|v\|}$ is a unit vector.

"Normalizing" length to 1 is replacing v by $\frac{v}{\|v\|}$.