Review for Exam 2:

Examples of maps which are or are not isomorphisms:

$L: V \to W$ is an isomorphism when L is **bijective**, that is, both inj and surj.

L is inj. iff $Ker(L) = \{\theta_V\}$.

L is surj. iff $Range(L) = W$.

$$\begin{tikzcd} V \arrow[r, "L"] \arrow[d, "{[\ ]_S}"] & W \arrow[d, "{[\ ]_T}"] \\ F^n \arrow[r, "{{}_T[L]_S}", "m\times n"'] & F^m \end{tikzcd}$$

Let $A = {}_T[L]_S \in F^m_n$

$[A \mid 0^m_1] \xrightarrow{r.r.} [C \mid 0^m_1]$

RREF with $r = rank(A)$ leading 1's (pivot columns).

L inj iff $r = n$ (no free variables)

To check if L is surj., look at when $AX = B$ is consistent! 218.2

$[A|B] \xrightarrow{r.r.} \underbrace{[C|D]}_{RREF}$

${}_T[L]_S [v]_S = [L(v)]_T$

is consistent for all $B \in F^m$ when $r = m$

<u>Conclusion</u>: L is <u>bij</u> iff $n = r = m$.

Say V is <u>isomorphic</u> to W when $\exists L: V \to W$ s.t. L is bijective (an isom.)

<u>Th</u>: For fin. dim'l V and W, $V \simeq W$ iff $\dim(V) = \dim(W)$.