

### Exam 3 Review:

If  $A \in F_n$  has distinct e-values  $\lambda_1, \dots, \lambda_r \in F$  and  $\text{Char}_A(\lambda) = \prod_{i=1}^r (\lambda - \lambda_i)^{k_i}$ ,  $k_i = \text{alg. mult. of } \lambda_i$  for  $A$ , but geom. mult. for  $\lambda_i$  is  $\dim(A_{\lambda_i}) = g_i$ .

Then we know: ①  $k_1 + \dots + k_r = n = \deg(\text{Char}_A(\lambda))$

② For  $1 \leq i \leq r$ ,  $1 \leq g_i \leq k_i$

③  $T = T_1 \cup \dots \cup T_r$  is indep set of e-vectors for  $A$

④  $T$  is a basis of  $F^n$  iff  $\langle T \rangle = F^n$  iff  $g_1 + g_2 + \dots + g_r = n$ .

If any  $g_i < k_i$  then  $T$  is not a basis of  $F^n$  so  $A$  is not diag-able.

⑤  $k_i = 1 \Rightarrow g_i = 1$

Note.  $\lambda_i \in F$  is an e-value for  $A \in F_n$  (300)

means  $A - \lambda_i I_n$  has rank  $< n$

iff  $[A - \lambda_i I_n | 0^n]$  has non-triv. solution,

$\text{Nul}(A - \lambda_i I_n)$  is the e-space  $A_{\lambda_i}$

$T_i$  is a basis of  $A_{\lambda_i}$ .