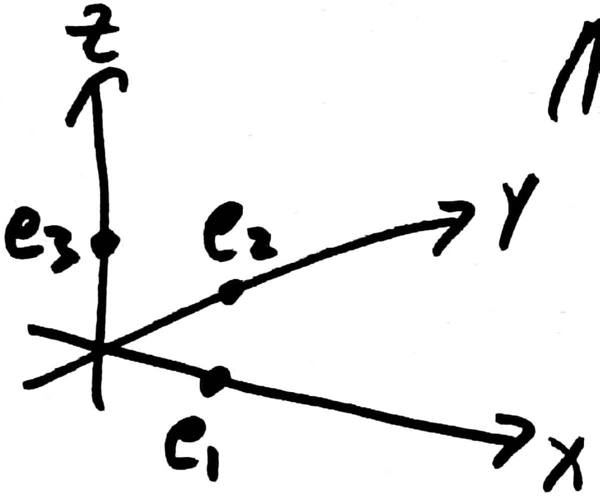
$\underset{\text{subset}}{S \subseteq V}$, $\underset{\text{span of } S}{\langle S\rangle} = \{\text{all lin. combos. from } S\} \underset{\text{subspace}}{\leq} V$

if $S = \{v_1, \dots, v_m\}$ then $\langle S\rangle = \left\{ \sum_{i=1}^{m} x_i v_i \in V \;\middle|\; x_i \in F \right\}$



$\mathbb{R}^3$

If $S = \left\{ e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

then $\langle e_1 \rangle = \{ x_1 e_1 \in \mathbb{R}^3 \mid x_1 \in \mathbb{R} \}$ = "x-axis"

$\langle e_2 \rangle$ = "y-axis"

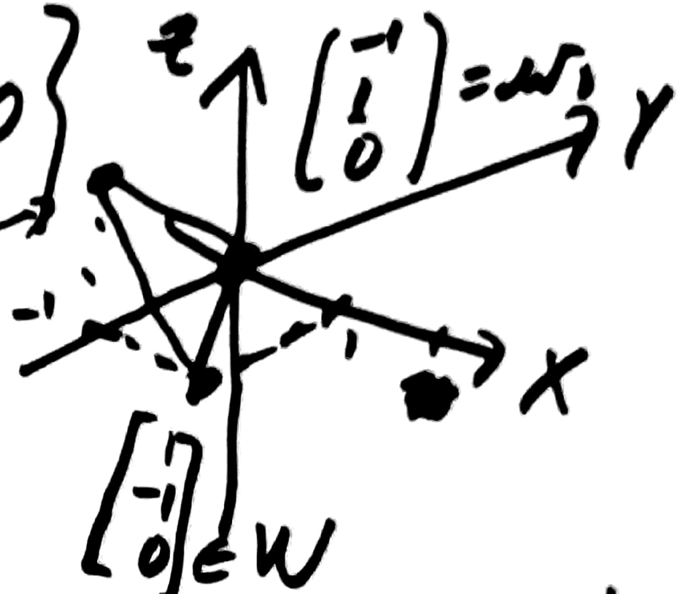
$\langle e_3 \rangle$ = "z-axis"

$\langle e_1, e_2 \rangle = \{ x_1 e_1 + x_2 e_2 \in \mathbb{R}^3 \mid x_1, x_2 \in \mathbb{R} \}$

$= \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_1, x_2 \in \mathbb{R} \right\}$ = "x-y plane"

$\boxed{\langle e_1, e_2, e_3 \rangle = \mathbb{R}^3}$

Let $W = \left\{ \begin{bmatrix} x\\ y\\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, x+y+z=0 \right\}$



solve

$[1\ 1\ 1 \mid 0]$ $\quad x = -y-z$

RREF $\quad y \in$ Free, $\quad z =$ Free $\quad \begin{bmatrix} 0\\ -1\\ 1 \end{bmatrix} \in W$

$$W = \left\{ \begin{bmatrix} -y-z\\ y\\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, y, z \in \mathbb{R} \right\} = \left\{ y \underbrace{\begin{bmatrix} -1\\ 1\\ 0 \end{bmatrix}}_{w_1} + z \underbrace{\begin{bmatrix} -1\\ 0\\ 1 \end{bmatrix}}_{w_2} \in \mathbb{R}^3 \,\middle|\, y, z \in \mathbb{R} \right\}$$

$$= \left\langle \underbrace{\begin{bmatrix} -1\\ 1\\ 0 \end{bmatrix}}_{w_1}, \underbrace{\begin{bmatrix} -1\\ 0\\ 1 \end{bmatrix}}_{w_2} \right\rangle$$ is the plane through $\begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}$ containing w_1 and w_2.

So $T = \{w_1, w_2\}$ spans W. is a basis of W. T is indep. since

$x_1 w_1 + x_2 w_2 = 0^3_1$ $\quad \left[\begin{array}{cc|c} -1 & -1 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0 \end{array}\right]$ $\quad x_1 = 0$, $\quad x_2 = 0$

$$\begin{tikzcd} y \in \mathbb{R}^p \arrow[r, "L_B"] \arrow[dr, "L_C = L_A \circ L_B"'] & \mathbb{R}^n \arrow[d, "L_A"] \\ & \mathbb{R}^m \end{tikzcd}$$

$(L_A \circ L_B)(y) = L_A(L_B(y)) = L_C(y)$

$L_B(y) = x \in \mathbb{R}^n$

defines unique $C \in \mathbb{R}^m_p$

$1 \le i \le m$

$1 \le j \le n$

$1 \le k \le p$

we say $\underset{m \times p}{C} = \underset{(m\times n)(n\times p)}{AB}$ matr. prod.

$C = [c_{ik}]$ where $c_{ik} = \sum_{j=1}^{n} a_{ij} \cdot b_{jk}$

$$[a_{i1}\ a_{i2} \cdots a_{in}] \begin{bmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{bmatrix}$$