NAME (Printed): ____

Math 304-6 Linear Algebra Spring 2025 Quiz 2 Feingold

INSTRUCTIONS: **Fill in the blank for each problem**. For all problems, let $A \in \mathbb{F}^m_n$ be an $m \times n$ matrix with entries from field $\mathbb{F}$, let $\mathbf{0} \in \mathbb{F}^m$ be the $m \times 1$ zero matrix, and let $L_A : \mathbb{F}^n \to \mathbb{F}^m$ be the function associated with A defined as $L_A(X) = AX$. Each fill-in blank is worth one point.

(1) If A row reduces to matrix C in RREF with r non-zero rows, then the homogeneous linear system $AX = \mathbf{0}$ has ____________ free variables from $\mathbb{F}$ in its solution.

(2) If the linear system $AX = \mathbf{0}$ has only the trivial solution, then $rank(A) =$ ____.

(3) If the linear system $AX = B$ is consistent for any $B \in \mathbb{F}^m$, then $rank(A) =$ ____.

(4) If $rank(A) = m$ then as a function L_A is ________________.

(5) If $rank(A) = n$ then as a function L_A is ________________.

(6) In general, for $A \in \mathbb{F}^m_n$, we only know that $rank(A) \leq$ ________________.

(7) If $m > n$ then as a function L_A **cannot be** ________________.

(8) If $m < n$ then as a function L_A **cannot be** ________________.

(9) If L_A is bijective (both injective and surjective) then ____ $= rank(A) =$ ____.

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INSTRUCTIONS: **Fill in the blank for each problem**. For all problems, let $A \in \mathbb{F}_n^m$ be an $m \times n$ matrix with entries from field $\mathbb{F}$, let $\mathbf{0} \in \mathbb{F}^m$ be the $m \times 1$ zero matrix, and let $L_A : \mathbb{F}^n \to \mathbb{F}^m$ be the function associated with A defined as $L_A(X) = AX$. Each fill-in blank is worth one point.

(1) If A row reduces to matrix C in RREF with r non-zero rows, then the homogeneous linear system $AX = \mathbf{0}$ has $\underline{n - r}$ free variables in its solution.

(2) If the linear system $AX = \mathbf{0}$ has only the trivial solution, then $rank(A) = \underline{n}$.

(3) If the linear system $AX = B$ is consistent for any $B \in \mathbb{F}^m$, then $rank(A) = \underline{m}$.

(4) If $rank(A) = m$ then then as a function L_A is $\underline{onto}$ or $\underline{surjective}$.

(5) If $rank(A) = n$ then then as a function L_A is $\underline{one - to - one}$ or $\underline{injective}$.

(6) In general, for $A \in \mathbb{F}_n^m$, we only know that $rank(A) \leq \underline{Min(m, n)}$.

(7) If $m > n$ then then as a function L_A **cannot be** $\underline{surjective}$ since $rank(A) = r \leq n < m$ so there are consistency conditions on B for $AX = B$.

(8) If $m < n$ then then as a function L_A **cannot be** $\underline{injective}$ since $rank(A) = r \leq m < n$ so there are $n - r > 0$ free variables in the solutions to $AX = \mathbf{0}$.

(9) If L_A is bijective (both injective and surjective) then $\underline{m} = rank(A) = \underline{n}$.