NAME (Printed): ____

Math 304-6 Linear Algebra Spring 2025 Quiz 3 Feingold

INSTRUCTIONS: Show all calculations needed to justify your answers.

Let $L_A : \mathbb{F}^2 \to \mathbb{F}^3$ and $L_B : \mathbb{F}^2 \to \mathbb{F}^2$ be the functions

$$L_A(X) = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \qquad \text{and} \qquad L_B(Y) = B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

(1) (2 points) Find the matrices A and B.

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(2) (2 points) Use the **formulas** for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$. **Do not just multiply the matrices A and B for this question.**

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(3) (2 points) Use the formula you got for $(L_A \circ L_B)(Y)$ in part 2 to find the matrix C such that $L_C = L_A \circ L_B$.

(4) (2 points) Compute the matrix product AB using $Row_i(A)Col_k(B)$ to get the (i, k)-entry of AB.

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(5) (2 points) What is the relation between your C in part (3) and your AB in part (4)? What should be the relation?

Math 304-6 Linear Algebra Spring 2025 Quiz 3 Solutions Feingold

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Let $L_A : \mathbb{F}^2 \to \mathbb{F}^3$ and $L_B : \mathbb{F}^2 \to \mathbb{F}^2$ be the functions

$$L_A(X) = L_A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \quad \text{and} \quad L_B(Y) = L_B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

(1) **Solution:** (2 points) $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ since

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 + 2x_2 \\ 2x_1 - x_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix}.$$

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(2) Use the formulas for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$. **Do not just multiply the matrices A and B for this question.**
Solution: (2 points)

$$(L_A \circ L_B) \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = L_A \begin{bmatrix} 2y_1 - y_2 \\ y_1 + 2y_2 \end{bmatrix} = \begin{bmatrix} (2y_1 - y_2) + (y_1 + 2y_2) \\ (2y_1 - y_2) + 2(y_1 + 2y_2) \\ 2(2y_1 - y_2) - (y_1 + 2y_2) \end{bmatrix} = \begin{bmatrix} 3y_1 + y_2 \\ 4y_1 + 3y_2 \\ 3y_1 - 4y_2 \end{bmatrix}$$

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(3) **Solution:** (2 points) $C = \begin{bmatrix} 3 & 1 \\ 4 & 3 \\ 3 & -4 \end{bmatrix}$ since $\begin{bmatrix} 3 & 1 \\ 4 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3y_1 + y_2 \\ 4y_1 + 3y_2 \\ 3y_1 - 4y_2 \end{bmatrix}$.

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(4) (2 points) Compute the matrix product AB using $Row_i(A)Col_k(B)$ to get the (i,k)-entry of AB.
Solution: (2 points)

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} (2+1) & (-1+2) \\ (2+2) & (-1+4) \\ (4-1) & (-2-2) \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 3 \\ 3 & -4 \end{bmatrix}.$$

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(5) (2 points) What is the relation between your C in part (3) and your AB in part (4)? What should be the relation?

Solution: (2 points) The relation between C in part (3) and AB in part (4) is that $C = AB$, which is what it should be since $L_A \circ L_B = L_{AB}$.
