NAME (Printed): ______________________________

Math 304-6 Linear Algebra Spring 2025 Quiz 8 Feingold

Show all work needed to justify your answers.

(1) (2 Pts) Compute $\det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

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(2) (2 Pts) If $\det(A) = 3$ for $A \in \mathbb{R}^5_5$, find $\det(2A)$.

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(3) (2 Pts) If $A \in \mathbb{R}^4_4$ with $Char_A(t) = (t-1)^3(t-2)$, what are all the possibilities for the geometric multiplicities $g_1 = \dim(A_1)$ and $g_2 = \dim(A_2)$.

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(4) (2 Pts) If $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, find $\det(A^2 B^T C^{-1})$ where B^T means B transpose.

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(5) (2 Pts) For $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, find a basis for the 0-eigenspace A_0.

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(1) (2 Pts) $\det\begin{bmatrix}1&2&1&2\\2&3&2&3\\3&2&3&2\\1&1&1&1\end{bmatrix} = \det\begin{bmatrix}1&2&1&2\\0&-1&0&-1\\0&-4&0&-4\\1&1&1&1\end{bmatrix} = \det\begin{bmatrix}1&2&1&2\\0&-1&0&-1\\0&0&0&0\\1&1&1&1\end{bmatrix} = 0$ by using elementary adder row operations to get a matrix with a zero row.

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(2) (2 Pts) If $\det(A) = 3$ for $A \in \mathbb{R}^5_5$, then $\det(2A) = 2^5\det(A) = (32)(3) = 96$ by factoring out a 2 from each of the five rows of $2A$.

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(3) (2 Pts) If $A \in \mathbb{R}^4_4$ with $Char_A(t) = (t-1)^3(t-2)$, then by the theorem which says $1 \le g_i \le k_i$, the possibilities are $1 \le g_1 \le 3$ and $g_2 = 1$ since the algebraic multiplicities are $k_1 = 3$ and $k_2 = 1$.

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(4) (2 Pts) If $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, then

$$\det(A^2B^TC^{-1}) = (\det(A))^2\det(B)(\det(C))^{-1} = \frac{(4)(3)}{5} = \frac{12}{5}.$$

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(5) (2 Pts) For $A = \begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$, the 0-eigenspace $A_0 = Nul(A) =$

$$\left\{\begin{bmatrix}-r-s-t\\r\\s\\t\end{bmatrix} \in \mathbb{R}^4 \mid r,s,t \in \mathbb{R}\right\} \text{ which has basis } \left\{\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\0\\1\end{bmatrix}\right\}.$$
