

NAME (Printed): _____

Math 304-6 Linear Algebra Spring 2026 Quiz 7 Feingold

Show all work needed to justify your answers.

Carry out the following steps to diagonalize $A = \begin{bmatrix} 12 & -5 \\ 30 & -13 \end{bmatrix}$.

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- (1) (2 Pts) Find $\det(A - tI_2) = \det \begin{bmatrix} 12 - t & -5 \\ 30 & -13 - t \end{bmatrix}$ and write it in factored form $(t - \lambda_1)(t - \lambda_2)$ to get the eigenvalues λ_1 and λ_2 .
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- (2) (2 Pts) Find a basis vector w_1 for the λ_1 -eigenspace by solving $[A - \lambda_1 I_2 | 0]$.
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- (3) (2 Pts) Find a basis vector w_2 for the λ_2 -eigenspace by solving $[A - \lambda_2 I_2 | 0]$.
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- (4) (2 Pts) Find the invertible matrix P whose columns are w_1 and w_2 , and find P^{-1} .
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- (5) (2 Pts) Compute $P^{-1}AP$ and verify that it is the diagonal matrix D with diagonal entries λ_1 and λ_2 .
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Show all work needed to justify your answers.

Carry out the following steps to diagonalize $A = \begin{bmatrix} 12 & -5 \\ 30 & -13 \end{bmatrix}$.

- (1) (2 Pts) $\det(A - tI_2) = \det \begin{bmatrix} 12-t & -5 \\ 30 & -13-t \end{bmatrix} = (12-t)(-13-t) - (-5)(30) = t^2 + t - 156 + 150 = t^2 + t - 6 = (t-2)(t+3)$ so the eigenvalues of A are $\lambda_1 = 2$ and $\lambda_2 = -3$.
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- (2) (2 Pts) The λ_1 -eigenspace is found by solving $[A - 2I_2|0]$. Row reduce

$$\left[\begin{array}{cc|c} 10 & -5 & 0 \\ 30 & -15 & 0 \end{array} \right] \text{ to } \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right] \text{ so } 2x_1 = x_2 \text{ has basis vector } w_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

- (3) (2 Pts) The λ_2 -eigenspace is found by solving $[A + 3I_2|0]$. Row reduce

$$\left[\begin{array}{cc|c} 15 & -5 & 0 \\ 30 & -10 & 0 \end{array} \right] \text{ to } \left[\begin{array}{cc|c} 3 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right] \text{ so } 3x_1 = x_2 \text{ has basis vector } w_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

- (4) (2 Pts) The invertible matrix $P = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$.
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- (5) (2 Pts)

$$P^{-1}AP = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 12 & -5 \\ 30 & -13 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

is the diagonal matrix D with diagonal entries λ_1 and λ_2 .
