

NAME (Printed): _____

Math 304-6 Linear Algebra Spring 2026 Quiz 8 Feingold

Show all work needed to justify your answers.

(1) (2 Pts) Compute $\det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

(2) (2 Pts) If $\det(A) = 3$ for $A \in \mathbb{R}_3^5$, find $\det(2A)$.

(3) (2 Pts) If $A \in \mathbb{R}_4^4$ with $\text{Char}_A(t) = (t - 1)^3(t - 2)$, what are all the possibilities for the geometric multiplicities $g_1 = \dim(A_1)$ and $g_2 = \dim(A_2)$.

(4) (2 Pts) If $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, find $\det(A^3 B^T C^{-1})$ where B^T means B transpose.

(5) (2 Pts) For $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, find a basis for the 0-eigenspace A_0 .

Show all work needed to justify your answers.

$$(1) \text{ (2 Pts) } \det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \det \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \det \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = 0 \text{ by}$$

using elementary adder row operations to get a matrix with a zero row. Another method is $\det(A) = \det(A^T) = 0$ because A^T has identical rows.

(2) (2 Pts) If $\det(A) = 3$ for $A \in \mathbb{R}_5^5$, then $\det(2A) = 2^5 \det(A) = (32)(3) = 96$ by factoring out a 2 from each of the five rows of $2A$.

(3) (2 Pts) If $A \in \mathbb{R}_4^4$ with $Char_A(t) = (t - 1)^3(t - 2)$, then by the theorem which says $1 \leq g_i \leq k_i$, the possibilities are $1 \leq g_1 \leq 3$ and $g_2 = 1$ since the algebraic multiplicities are $k_1 = 3$ and $k_2 = 1$.

(4) (2 Pts) If $\det(A) = 2$, $\det(B) = 3$ and $\det(C) = 5$, then

$$\det(A^3 B^T C^{-1}) = (\det(A))^3 \det(B) (\det(C))^{-1} = \frac{(8)(3)}{5} = \frac{24}{5}.$$

(5) (2 Pts) For $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, the 0-eigenspace $A_0 = Nul(A) =$

$$\left\{ \begin{bmatrix} -r - s - t \\ r \\ s \\ t \end{bmatrix} \in \mathbb{R}^4 \mid r, s, t \in \mathbb{R} \right\} \text{ which has basis } \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$