Math 330-3 Number Systems Fall 2022 Quiz 1 Feingold

SHOW ALL WORK NECESSARY TO JUSTIFY YOUR ANSWERS. $\mathbb{N}^+ = \mathbb{N}\backslash\{0\}$

1. (15 Points) Define the sets

$$A = \{n \in \mathbb{Z} \mid gcd(3,n) = 1\} \qquad B = \{m \in \mathbb{Z} \mid m \equiv 0 \ (mod\ 3)\},$$

$$C = \{m \in \mathbb{Z} \mid m \equiv 1 \ (mod\ 3)\} \qquad D = \{m \in \mathbb{Z} \mid m \equiv 2 \ (mod\ 3)\}.$$

Determine whether each assertion below is true or false. If it is true, prove it. If it is false, show why. You may use the Division Algorithm and your knowledge of *gcd* in your answers.

(a) $A = B \cup C$. (b) $A = C \cup D$. (c) $\mathbb{Z} = B \cup C \cup D$.

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Solutions:

(a) $A = B \cup C$. False. $3 \in B$ since $3 \equiv 0 \pmod 3$, but $3 \notin A$ since $gcd(3,3) = 3$.

(b) $A = C \cup D$. True. Since $gcd(3,n) = 1$ iff $3 \nmid n$ iff $n \not\equiv 0 \pmod 3$ iff $n \equiv 1 \pmod 3$ or $n \equiv 2 \pmod 3$ iff $n \in C$ or $n \in D$ iff $n \in C \cup D$.

(c) $\mathbb{Z} = B \cup C \cup D$. True. By their definitions, $B, C, D \subseteq \mathbb{Z}$ so $B \cup C \cup D \subseteq \mathbb{Z}$. Show the reverse containment to get equality. By the Division Algorithm, for any $m \in \mathbb{Z}$ there exist integers q and r such that $m = 3q + r$ and $0 \leq r \leq 2$, so that $m \equiv r \pmod 3$. $r = 0$ means $m \in B$, $r = 1$ means $m \in C$, $r = 2$ means $m \in D$. Since $r = 0$ or $r = 1$ or $r = 2$, we get $m \in B$ or $m \in C$ or $m \in D$, so in any case, $m \in B \cup C \cup D$, so $\mathbb{Z} \subseteq B \cup C \cup D$.
