NAME (Printed): ______________________________

Math 330-3 Number Systems Fall 2022 Quiz 3 Feingold

SHOW ALL WORK NECESSARY TO JUSTIFY YOUR ANSWERS. $\mathbb{N}^+ = \mathbb{N}\setminus\{0\}$

Use the binomial theorem, $(x+y)^m = \sum_{i=0}^{m} \binom{m}{i} x^{m-i} y^i = x^m \sum_{i=0}^{m} \binom{m}{i} \left(\frac{y}{x}\right)^i$, to write each of the following sums as a **power of a rational number**.

1. (5 points) $\sum_{i=0}^{20} \binom{20}{i} \left(\frac{2}{3}\right)^i$

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2. (5 points) $\sum_{i=0}^{100} \binom{100}{i} \left(\frac{-4}{5}\right)^i$

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3. (5 points) Find the least $n \in \mathbb{N}^+$ such that $2^n \equiv 1 \pmod{11}$.

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4. (5 points) Find $0 \leq c < 11$ such that $2^{1563} \equiv c \pmod{11}$.

Math 330-3 Number Systems Fall 2022 Quiz 3 Solutions Feingold

Use the binomial theorem, $(x+y)^m = \sum_{i=0}^{m} \binom{m}{i} x^{m-i} y^i = x^m \sum_{i=0}^{m} \binom{m}{i} \left(\frac{y}{x}\right)^i$, to write each of the following sums as a **power of a rational number**.

1. (5 points) $\sum_{i=0}^{20} \binom{20}{i} \left(\frac{2}{3}\right)^i = \frac{(3+2)^{20}}{3^{20}} = \left(\frac{5}{3}\right)^{20}$
   using $x = 3$, $y = 2$ and $m = 20$ in the binomial theorem after dividing both sides by x^m.

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2. (5 points) $\sum_{i=0}^{100} \binom{100}{i} \left(\frac{-4}{5}\right)^i = \frac{(5-4)^{100}}{5^{100}} = \left(\frac{1}{5}\right)^{100}$
   using $x = 5$, $y = -4$ and $m = 100$ in the binomial theorem after dividing both sides by x^m.

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3. (5 points) Find the least $n \in \mathbb{N}^+$ such that $2^n \equiv 1 \pmod{11}$.
   Computing the positive powers $2^i \pmod{11}$ we find: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16 \equiv 5$, $2^5 \equiv 10$, $2^6 \equiv 20 \equiv 9$, $2^7 \equiv 18 \equiv 7$, $2^8 \equiv 14 \equiv 3$, $2^9 \equiv 6$, $2^{10} \equiv 12 \equiv 1$.
   So $n = 10$ is the least $n \in \mathbb{N}^+$ such that $2^n \equiv 1 \pmod{11}$.

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4. (5 points) Find $0 \le c < 11$ such that $2^{1563} \equiv c \pmod{11}$.
   Since $2^{10} \equiv 1 \pmod{11}$, we should write $1563 = (10)(156) + 3$ and compute

   $$2^{1563} \equiv 2^{(10)(156)+3} \equiv (2^{10})^{156}\, 2^3 \equiv 1^{156}\, 8 \equiv 8 \pmod{11}$$

   so $c = 8$.
