

NAME (Printed): _____

Math 330-3 Number Systems Fall 2022 Quiz 5 Feingold

SHOW ALL WORK NECESSARY TO JUSTIFY YOUR ANSWERS. $\mathbb{N}^+ = \mathbb{N} \setminus \{0\}$

Let $f : \mathbb{Q} \rightarrow \mathbb{Q}$ and $g : \mathbb{Q} \rightarrow \mathbb{Q}$ be the **bijective** functions defined by the following formulas, $\forall x \in \mathbb{Q}$, $f(x) = 2x + 3$ and $g(x) = -3x + 4$.

1. (5 points) Find formulas $x = f^{-1}(y)$ and $x = g^{-1}(y)$ for the inverse functions $f^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$ and $g^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$.

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2. (5 points) Find formulas for the compositions $f \circ g$ and $g \circ f$.

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3. (5 points) Find a formula for the inverse function $(f \circ g)^{-1}$.

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4. (5 points) Find a formula for the composition $g^{-1} \circ f^{-1}$ and compare it to your answer to (3).

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Let $f : \mathbb{Q} \rightarrow \mathbb{Q}$ and $g : \mathbb{Q} \rightarrow \mathbb{Q}$ be the **bijective** functions defined by the following formulas, $\forall x \in \mathbb{Q}$, $f(x) = 2x + 3$ and $g(x) = -3x + 4$.

1. (5 points) Find formulas $x = f^{-1}(y)$ and $x = g^{-1}(y)$ for the inverse functions $f^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$ and $g^{-1} : \mathbb{Q} \rightarrow \mathbb{Q}$.

Solution: Since $y = f(x) = 2x + 3$ iff $x = \frac{1}{2}(y - 3)$ we have

$$x = f^{-1}(y) = \frac{1}{2}(y - 3)$$

is the formula for the inverse of f . Since $y = g(x) = -3x + 4$ iff $x = \frac{-1}{3}(y - 4)$ we have

$$x = g^{-1}(y) = \frac{-1}{3}(y - 4)$$

is the formula for the inverse of g .

2. (5 points) Find formulas for the compositions $f \circ g$ and $g \circ f$.

Solution:

$$(f \circ g)(x) = f(g(x)) = f(-3x + 4) = 2(-3x + 4) + 3 = -6x + 11$$

and

$$(g \circ f)(x) = g(f(x)) = g(2x + 3) = -3(2x + 3) + 4 = -6x - 5.$$

3. (5 points) Find a formula for the inverse function $(f \circ g)^{-1}$.

Solution: Since $y = (f \circ g)(x) = -6x + 11$ iff $x = \frac{-1}{6}(y - 11)$ we have

$$x = (f \circ g)^{-1}(y) = \frac{-1}{6}(y - 11)$$

is the formula for the inverse of $f \circ g$.

4. (5 points) Find a formula for the composition $g^{-1} \circ f^{-1}$ and compare it to your answer to (3).

Solution: We have

$$\begin{aligned}(g^{-1} \circ f^{-1})(y) &= g^{-1}(f^{-1}(y)) = g^{-1}\left(\frac{1}{2}(y - 3)\right) = \frac{-1}{3}\left(\frac{1}{2}(y - 3) - 4\right) \\ &= \frac{-1}{6}(y - 3) + \frac{4}{3} = \frac{-1}{6}y + \frac{3}{6} + \frac{8}{6} = \frac{-1}{6}y + \frac{11}{6} = \frac{-1}{6}(y - 11)\end{aligned}$$

which equals $(f \circ g)^{-1}(y)$ from part (3).
