

NAME (Printed): \_\_\_\_\_

Math 404    Advanced Linear Algebra    Spring 2023    Quiz 2    Feingold

**INSTRUCTIONS: For each problem fill in the blank or circle your choice of the most accurate answer.** For all problems, let  $A \in \mathbf{F}_n^m$  be an  $m \times n$  matrix of numbers from field  $\mathbf{F}$ , let  $\mathbf{0} \in \mathbf{F}^m$  be the  $m \times 1$  zero matrix, and let  $L_A : \mathbf{F}^n \rightarrow \mathbf{F}^m$  be the function associated with  $A$  defined as  $L_A(X) = AX$ . Each answer is worth one point.

- (1) If  $A$  row reduces to matrix  $C$  in RREF with  $r$  non-zero rows, then the homogeneous linear system  $AX = \mathbf{0}$  has exactly \_\_\_\_\_ free variables in its solution.
- (2) If the linear system  $AX = \mathbf{0}$  has a non-trivial solution, then  $\text{rank}(A)$  \_\_\_\_.
- (3) If  $\text{rank}(A) = m$  then as a function  $L_A$  is (**circle correct answer**)  
incomprehensible   inflammable   invertible   injective   surjective   bijective.
- (4) In general, for  $A \in \mathbf{F}_n^m$ , we only know that  $\text{rank}(A) \leq$  \_\_\_\_\_.
- (5) If  $m < n$  then as a function  $L_A : \mathbf{F}^n \rightarrow \mathbf{F}^m$  **cannot be (circle correct answer)**  
defined   injective   surjective

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- (1) If  $A$  row reduces to matrix  $C$  in RREF with  $r$  non-zero rows, then the homogeneous linear system  $AX = \mathbf{0}$  has exactly  $n - r$  free variables in its solution.
- (2) If the linear system  $AX = \mathbf{0}$  has a non-trivial solution, then  $\text{rank}(A) \leq \underline{n}$ .
- (3) If  $\text{rank}(A) = m$  then then as a function  $L_A$  is surjective.
- (4) In general, for  $A \in \mathbf{F}_n^m$ , we only know that  $\text{rank}(A) \leq \underline{\text{Min}(m, n)}$ .
- (5) If  $m < n$  then then as a function  $L_A$  **cannot be** injective since  $\text{rank}(A) = r \leq m < n$  so there are  $n - r > 0$  free variables in the solutions to  $AX = \mathbf{0}$ .