

NAME (Printed): _____

Math 404 Advanced Linear Algebra Spring 2023 Quiz 4 Feingold

Show all calculations and reasons needed to justify your answers.

For the real matrix $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ do the following.

- (1) (3 Points) Find the **characteristic polynomial** $\det(tI_4 - A) = \det(A - tI_4)$.
 - (2) (1 Point) Find the **eigenvalues** of A .
 - (3) (1 Point) Find the **algebraic multiplicity** of each eigenvalue of A .
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Show all calculations and reasons needed to justify your answers.

(1) (3 Points) $\det(tI_4 - A) = \det(A - tI_4) =$

$$\det \begin{bmatrix} -t & 1 & 0 & 1 \\ 1 & -t & 1 & 0 \\ 0 & 1 & -t & 1 \\ 1 & 0 & 1 & -t \end{bmatrix} = \det \begin{bmatrix} 0 & 1 & t & (1-t^2) \\ 0 & -t & 0 & t \\ 0 & 1 & -t & 1 \\ 1 & 0 & 1 & -t \end{bmatrix} = -\det \begin{bmatrix} 1 & t & (1-t^2) \\ -t & 0 & t \\ 1 & -t & 1 \end{bmatrix} =$$

$$t \det \begin{bmatrix} 1 & t & (1-t^2) \\ 1 & 0 & -1 \\ 1 & -t & 1 \end{bmatrix} = t \det \begin{bmatrix} 0 & t & (2-t^2) \\ 1 & 0 & -1 \\ 0 & -t & 2 \end{bmatrix} = t \det \begin{bmatrix} 0 & 0 & (4-t^2) \\ 1 & 0 & -1 \\ 0 & -t & 2 \end{bmatrix} =$$

$$t \det \begin{bmatrix} 1 & 0 & -1 \\ 0 & -t & 2 \\ 0 & 0 & (4-t^2) \end{bmatrix} = t^2 (t^2 - 4) = t^2 (t - 2) (t + 2)$$

(2) (1 Point) The eigenvalues are $\lambda_1 = 0$, $\lambda_2 = 2$ and $\lambda_3 = -2$.

(3) (1 Point) The corresponding algebraic multiplicities of those eigenvalues are: $k_1 = 2$, $k_2 = 1$ and $k_3 = 1$.
