

NAME (Printed): _____

Show all calculations and reasons needed to justify your answers.

Math 404 Advanced Linear Algebra Spring 2023 Quiz 6 Feingold

(1) (5 points) Suppose that $A \in \mathbb{R}_7^7$ has **characteristic** and **minimal** polynomials

$$\text{char}_A(t) = (t - 5)^7 \quad \text{and} \quad m_A(t) = (t - 5)^3.$$

Find all possible **Jordan canonical form** matrices J to which A might be similar, but not to each other, and **for each one give the geometric multiplicity** of the eigenvalue $\lambda_1 = 5$.

Show all calculations and reasons needed to justify your answers.

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Solution: $\text{char}_A(t) = (t - 5)^7$ and $m_A(t) = (t - 5)^3$ so there is only one eigenvalue, $\lambda_1 = 5$ with algebraic multiplicity $k_1 = 7$. The power in the minimal polynomial $m_1 = 3$ tells the size of the largest Jordan block. Let

$$B = J(5, 3) = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix} \quad \text{and} \quad C = J(5, 2) = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$$

then there are four possible Jordan canonical form matrices similar to A , corresponding to the partitions of 7 into parts with largest part 3:

<i>Partition :</i>	$3 + 3 + 1$	$3 + 2 + 2$	$3 + 2 + 1 + 1$	$3 + 1 + 1 + 1 + 1$
<i>J - Form :</i>	$\text{Diag}(B, B, 5)$	$\text{Diag}(B, C, C)$	$\text{Diag}(B, C, 5, 5)$	$\text{Diag}(B, 5, 5, 5, 5)$
<i>Geom. Mult :</i>	3	3	4	5

where the corresponding geometric multiplicity is the number of Jordan blocks.
