

NAME (Printed): _____

Math 404 Advanced Linear Algebra Spring 2023 Quiz 8 Feingold

Show all calculations and reasons needed to justify your answers.

Let $V = \mathbf{R}^5$ with the standard dot product. Let $W = \langle T \rangle$ where

$$T = \left\{ w_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, w_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ is an ordered basis of } W.$$

- (1) (5 Pts) Use the Gram-Schmidt process to convert T into an **orthogonal** basis $T' = \{w'_1 = w_1, w'_2, w'_3\}$ for W . **Rescale w'_3 to avoid fractions in your final answer.**
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Solution: Step 1: $w'_1 = w_1$.

$$\text{Step 2: } w'_2 = w_2 - \left(\frac{w_2 \cdot w'_1}{w'_1 \cdot w'_1} \right) w'_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} - \frac{15}{5} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}.$$

$$\text{Step 3: } w'_3 = w_3 - \left(\frac{w_3 \cdot w'_1}{w'_1 \cdot w'_1} \right) w'_1 - \left(\frac{w_3 \cdot w'_2}{w'_2 \cdot w'_2} \right) w'_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{-1}{5} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{5}{10} \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10} \left(\begin{bmatrix} -10 \\ -10 \\ 0 \\ 0 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \\ 0 \\ -5 \\ -10 \end{bmatrix} \right) = \frac{1}{10} \begin{bmatrix} 2 \\ -3 \\ 2 \\ -3 \\ 2 \end{bmatrix}.$$

$$\text{So, rescaling } w'_3 \text{ to avoid fractions, } T' = \left\{ w'_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, w'_2 = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, w'_3 = \begin{bmatrix} 2 \\ -3 \\ 2 \\ -3 \\ 2 \end{bmatrix} \right\}.$$

By the process, $\langle T' \rangle = \langle T \rangle = W$, and we check that $w'_i \cdot w'_j = 0$ for $1 \leq i < j \leq 3$:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = 0, \quad \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 2 \\ -3 \\ 2 \end{bmatrix} = 0, \quad \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 2 \\ -3 \\ 2 \end{bmatrix} = 0.$$
