NAME (Printed): ______________________________

Math 507 Linear Algebra and Matrix Theory Fall 2025 Quiz 1 Feingold

Show all calculations needed to justify your answers. A^T **means** A **transpose.**

Let $L : \mathbb{F}_2^2 \to \mathbb{F}_2^2$ be the function $L \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & b-c \\ 0 & 0 \end{bmatrix}$.

(1) (1 pt) For $A_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$, show $L(A_1 + A_2) = L(A_1) + L(A_2)$.

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(2) (1 pt) For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $r \in \mathbb{F}$, show that $L(rA) = rL(A)$.

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(3) (2 pts) Parts (1) and (2) say that L is linear. Find the subspace $Ker(L)$.

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(4) (1 pt) How does part (3) show that the set of all **symmetric** matrices, $Sym(\mathbb{F}_2^2) = \{A \in \mathbb{F}_2^2 \mid A^T = A\}$, is a subspace without directly checking closure of $Sym(\mathbb{F}_2^2)$ under $+$ and $\cdot$ and $0_2^2 \in Sym(\mathbb{F}_2^2)$?

Math 507 Linear Algebra and Matrix Theory Fall 2025 Quiz 1 Solutions Feingold

Show all calculations needed to justify your answers.

Let $L : \mathbb{F}_2^2 \to \mathbb{F}_2^2$ be the function $L\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & b-c \\ 0 & 0 \end{bmatrix}$.

(1) (1 pt) For $A_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$, show $L(A_1 + A_2) = L(A_1) + L(A_2)$.

Solution: $A_1 + A_2 = \begin{bmatrix} (a_1 + a_2) & (b_1 + b_2) \\ (c_1 + c_2) & (d_1 + d_2) \end{bmatrix}$ so

$$L(A_1+A_2) = \begin{bmatrix} 0 & (b_1 + b_2) - (c_1 + c_2) \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & (b_1 - c_1) \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & (b_2 - c_2) \\ 0 & 0 \end{bmatrix} = L(A_1)+L(A_2).$$

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(2) (1 pt) For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $r \in \mathbb{F}$, show that $L(rA) = rL(A)$.

Solution: $L(rA) = L\begin{bmatrix} ra & rb \\ rc & rd \end{bmatrix} = \begin{bmatrix} 0 & rb - rc \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & r(b-c) \\ 0 & 0 \end{bmatrix} = r\begin{bmatrix} 0 & b-c \\ 0 & 0 \end{bmatrix} = rL(A)$.

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(3) (2 pts) Parts (1) and (2) say that L is linear. Find the subspace $Ker(L)$.

Solution:

$$Ker(L) = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{F}_2^2 \,\middle|\, L(A) = \begin{bmatrix} 0 & b-c \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} =$$

$$\left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{F}_2^2 \,\middle|\, b = c \right\} = \left\{ A = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \in \mathbb{F}_2^2 \,\middle|\, a, b, d \in \mathbb{F} \right\} = \{A \in \mathbb{F}_2^2 \mid A^T = A\}.$$

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(4) (1 pt) How does part (3) show that the set of all **symmetric** matrices, $Sym(\mathbb{F}_2^2) = \{A \in \mathbb{F}_2^2 \mid A^T = A\}$, is a subspace without directly checking closure of $Sym(\mathbb{F}_2^2)$ under $+$ and $\cdot$ and $0_2^2 \in Sym(\mathbb{F}_2^2)$?

Solution: In part (3) we found that $Ker(L) = \{A \in \mathbb{F}_2^2 \mid A^T = A\}$ is the set of all **symmetric** matrices in $\mathbb{F}_2^2$, so as a kernel of a linear map it is a subspace by a theorem proved in class.
