NAME (Printed): ____

Math 507 Advanced Linear Algebra Fall 2025 Quiz 2 Feingold

INSTRUCTIONS: **For each problem fill in the blank or circle your choice of the most accurate answer**. For all problems, let $A \in \mathbb{F}^m_n$ be an $m \times n$ matrix of numbers from field $\mathbb{F}$, let $\mathbf{0} \in \mathbb{F}^m$ be the $m \times 1$ zero matrix, and let $L_A : \mathbb{F}^n \to \mathbb{F}^m$ be the function associated with A defined as $L_A(X) = AX$. Each answer is worth one point. No justifications of answers are needed.

(1) If A row reduces to matrix C in RREF with r non-zero rows, then the homogeneous linear system $AX = \mathbf{0}$ has exactly ____________ free variables in its solution.

(2) If the linear system $AX = \mathbf{0}$ has a non-trivial solution, then $rank(A)$ ____.

(3) If $rank(A) = m$ then as a function L_A is (**circle correct answer**)
$\underline{incomprehensible}$ $\underline{inflamable}$ $\underline{invertible}$ $\underline{injective}$ $\underline{surjective}$ $\underline{bijective}$.

(4) In general, for $A \in \mathbb{F}^m_n$, we only know that $rank(A) \leq$ ________________.

(5) If $m < n$ then as a function $L_A : \mathbb{F}^n \to \mathbb{F}^m$ **cannot be** (**circle correct answer**)
$\underline{defined}$ $\underline{injective}$ $\underline{surjective}$

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(1) If A row reduces to matrix C in RREF with r non-zero rows, then the homogeneous linear system $AX = \mathbf{0}$ has exactly $\underline{n - r}$ free variables in its solution.

(2) If the linear system $AX = \mathbf{0}$ has a non-trivial solution, then $rank(A)\underline{< n}$.

(3) If $rank(A) = m$ then then as a function L_A is $\underline{surjective}$.

(4) In general, for $A \in \mathbb{F}^m_n$, we only know that $rank(A) \leq \underline{Min(m, n)}$.

(5) If $m < n$ then then as a function L_A **cannot be** $\underline{injective}$ since $rank(A) = r \leq m < n$ so there are $n - r > 0$ free variables in the solutions to $AX = \mathbf{0}$.