NAME (Printed): ____

Math 507 Advanced Linear Algebra Fall 2025 Quiz 3 Feingold

Show all calculations needed to justify your answers.

Let $L_A : \mathbb{F}^3 \to \mathbb{F}^2$ and $L_B : \mathbb{F}^2 \to \mathbb{F}^3$ be the functions

$$L_A(X) = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 \\ 2x_1 + x_2 - x_3 \end{bmatrix} \qquad \text{and} \qquad L_B(Y) = B \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ 2y_1 + 3y_2 \end{bmatrix}.$$

(1) (1 point) Find the matrices A and B.

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(2) (1 point) Use the **formulas** for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$. **Do not just multiply matrices to get your answer. Show the algebra needed to do the composition of functions.**

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(3) (1 point) Use the formula you got for $(L_A \circ L_B)(Y)$ in part 2 to find the matrix C such that $L_C = L_A \circ L_B$.

(4) (1 points) Compute the matrix product AB.

(5) (1 point) What is the relation between your C in part (3) and your AB in part (4)? What should be the relation?

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$$L_A(X) = A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 \\ 2x_1 + x_2 - x_3 \end{bmatrix} \quad \text{and} \quad L_B(Y) = B\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ 2y_1 + 3y_2 \end{bmatrix}.$$

(1) (1 point) Find the matrices A and B.

Solution: $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}$ since

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 \\ 2x_1 + x_2 - x_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ 2y_1 + 3y_2 \end{bmatrix}.$$

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(2) (1 point) Use the **formulas** for L_A and L_B to get the **formula** for the composition $(L_A \circ L_B)(Y)$. **Do not just multiply matrices to get your answer. Show the algebra needed to do the composition of functions.**

Solution:

$$(L_A \circ L_B)\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = L_A\begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ 2y_1 + 3y_2 \end{bmatrix} = \begin{bmatrix} (y_1 + y_2) - (y_1 - y_2) + (2y_1 + 3y_2) \\ 2(y_1 + y_2) + (y_1 - y_2) - (2y_1 + 3y_2) \end{bmatrix} = \begin{bmatrix} 2y_1 + 5y_2 \\ y_1 - 2y_2 \end{bmatrix}$$

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(3) (1 point) Use the formula you got for $(L_A \circ L_B)(Y)$ in part 2 to find the matrix C such that $L_C = L_A \circ L_B$.

Solution: $C = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$ since $\begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2y_1 + 5y_2 \\ y_1 - 2y_2 \end{bmatrix}$.

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(4) (1 point) Compute the matrix product AB.

Solution:

$$AB = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} (1 - 1 + 2) & (1 + 1 + 3) \\ (2 + 1 - 2) & (2 - 1 - 3) \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}.$$

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(5) (1 point) What is the relation between your C in part (3) and your AB in part (4)? What should be the relation?

Solution: The relation between C in part (3) and AB in part (4) is that $C = AB$, which is what it should be since $L_A \circ L_B = L_{AB}$.
