NAME (Printed): ____

Show all calculations and reasons needed to justify your answers.

Math 507 Linear Algebra and Matrix Theory Fall 2025 Quiz 6 Feingold

(1) (5 points) Suppose that $A \in \mathbb{R}^7_7$ has **characteristic** and **minimal** polynomials

$$char_A(t) = (t-5)^7 \quad \text{and} \quad m_A(t) = (t-5)^3.$$

Find all possible **Jordan canonical form** matrices J to which A might be similar, but not to each other, and **for each one give the geometric multiplicity** of the eigenvalue $\lambda_1 = 5$. Use the notation $J(\lambda, n)$ for a basic Jordan block of size n with eigenvalue λ.

Show all calculations and reasons needed to justify your answers.

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Solution: $char_A(t) = (t-5)^7$ and $m_A(t) = (t-5)^3$ so there is only one eigenvalue, $\lambda_1 = 5$ with algebraic multiplicity $k_1 = 7$. The power in the minimal polynomial $m_1 = 3$ tells the size of the largest Jordan block. Let

$$B = J(5,3) = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix} \qquad \text{and} \qquad C = J(5,2) = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$$

then there are four possible Jordan canonical form matrices similar to A, corresponding to the partitions of 7 into parts with largest part 3:

$Partition:$	$3+3+1$	$3+2+2$	$3+2+1+1$	$3+1+1+1+1$
$J-Form:$	$Diag(B,B,5)$	$Diag(B,C,C)$	$Diag(B,C,5,5)$	$Diag(B,5,5,5,5)$
$Geom.Mult:$	3	3	4	5

where the corresponding geometric multiplicity is the number of Jordan blocks.
