<u>Webwork Question:</u> For what values of K does $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & -3 & -1 \\ K & 0 & 0 \end{bmatrix}$ have 3 <u>distinct</u> e-values?

$$|A - \lambda I_3| = \begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & -3-\lambda & -1 \\ K & 0 & -\lambda \end{vmatrix} = \left(\begin{array}{l}\text{by cofactor} \\ \text{expansion along} \\ \text{row 3}\end{array}\right)$$

$$= K \begin{vmatrix} -1 & 0 \\ (-3-\lambda) & -1 \end{vmatrix} + (-\lambda) \begin{vmatrix} 1-\lambda & -1 \\ -1 & -3-\lambda \end{vmatrix}$$

$$= K - \lambda\big[(\lambda-1)(\lambda+3) - 1\big) = K - \lambda(\lambda^2 + 2\lambda - 4) \text{ so}$$

$$\mathrm{Char}_A(\lambda) = \lambda^3 + 2\lambda^2 - 4\lambda - K = f(\lambda)$$

when does cubic poly. $f(\lambda)$ have 3 distinct roots? Calculus graphing problem.

<u>Note</u>: $f'(\lambda) = 3\lambda^2 + 4\lambda - 4 = (3\lambda - 2)(\lambda + 2)$

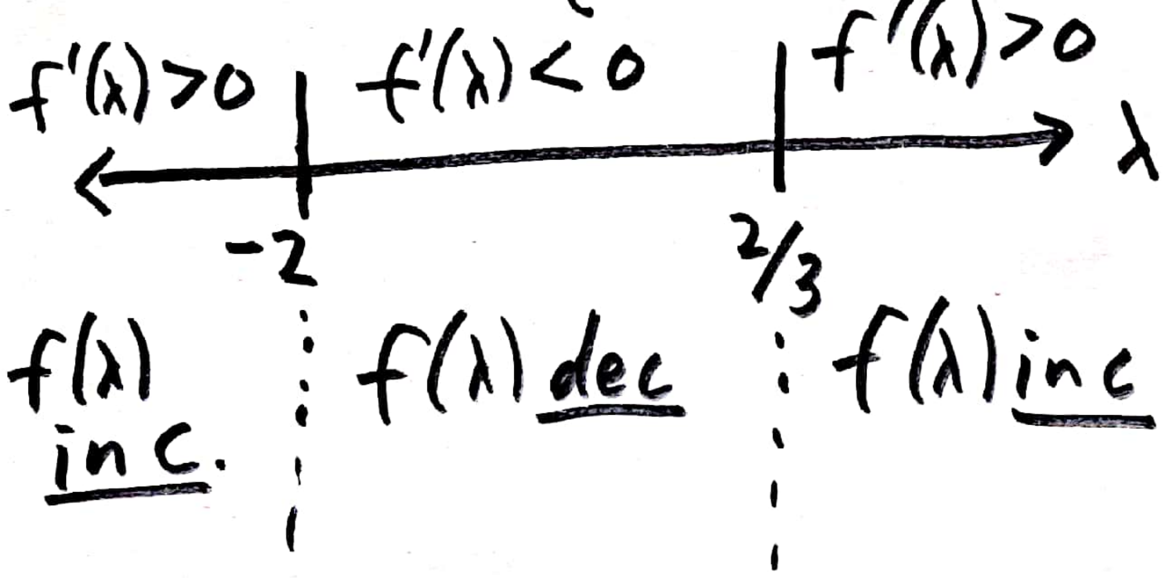
has roots $\lambda_1 = \frac{2}{3}$ and $\lambda_2 = -2$.

$f(\lambda)$ is increasing where $f''(\lambda) > 0$.

$f(\lambda)$ is decreasing where $f'(\lambda) < 0$.

$$3\lambda - 2 \text{ is } \begin{cases} < 0 & \text{when } \lambda < 2/3 \\ > 0 & \text{when } \lambda > 2/3 \end{cases}$$

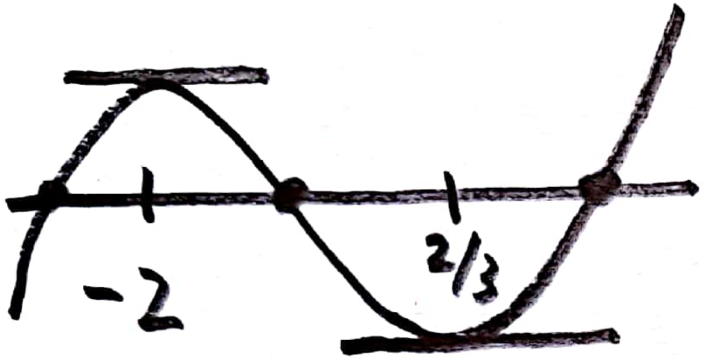
$$\lambda + 2 \text{ is } \begin{cases} < 0 & \text{when } \lambda < -2 \\ > 0 & \text{when } \lambda > -2 \end{cases} \quad \text{so}$$



so $f(\lambda)$ is

$$\begin{cases} \text{inc for } \lambda < -2 \\ \text{dec for } -2 < \lambda < \frac{2}{3} \\ \text{inc for } \frac{2}{3} < \lambda \end{cases}$$

The roots of $f'(\lambda)$, $\lambda_1 = \frac{2}{3}$ and $\lambda_2 = -2$, are the local max/min points of $f(\lambda)$.



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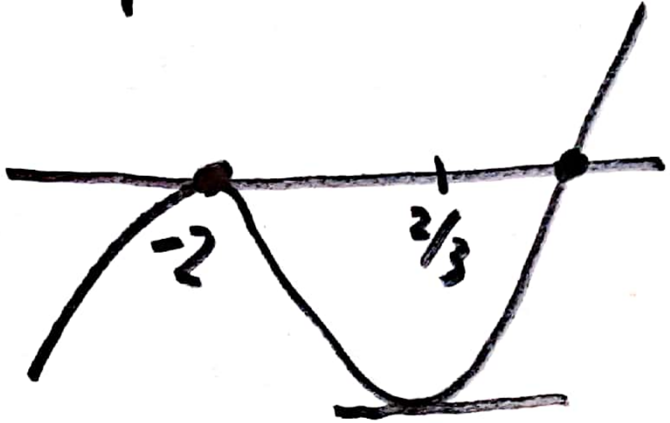

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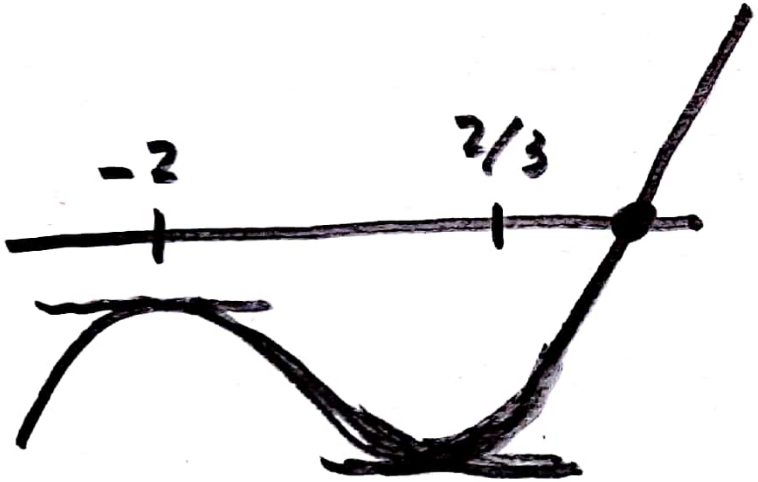

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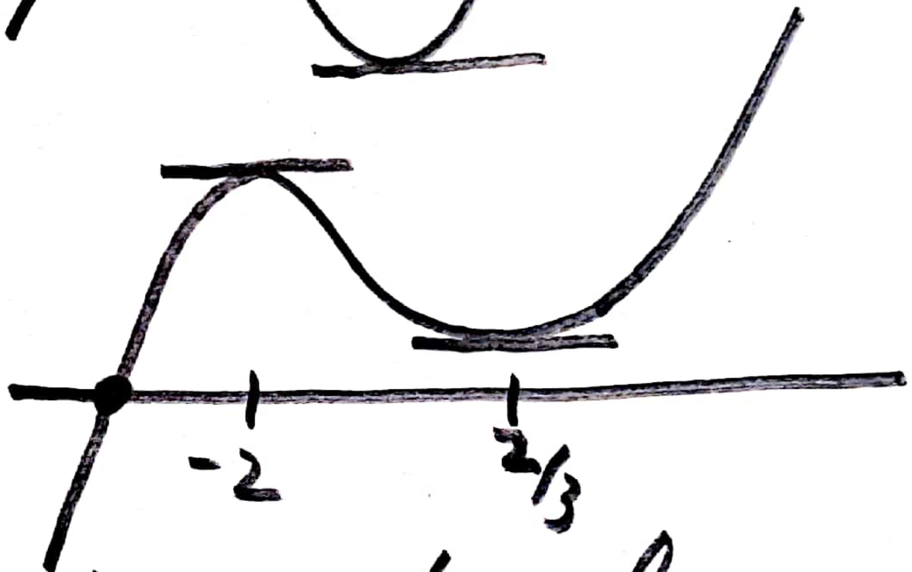

Basic shape is same, but vertical position depends on K.

To get vertical position, compute:

$f(-2) = (-2)^3 + 2(-2)^2 - 4(-2) - K = 8 - K$ and

$f(\frac{2}{3}) = (\frac{2}{3})^3 + 2(\frac{2}{3})^2 - 4(\frac{2}{3}) - K = \frac{8}{27} + \frac{8}{9} - \frac{8}{3} - K$

$= \frac{8 + 24 - 72}{27} - K = \frac{-40}{27} - K$

Since $\frac{-40}{27} - K < 8 - K$ get $f(\frac{2}{3}) < f(-2)$.

From graphs, only get 3 distinct roots when

$0 < f(-2)$ and $f(\frac{2}{3}) < 0$ so

$0 < 8 - K$ and $\frac{-40}{27} - K < 0$ so

$K < 8$ and $\frac{-40}{27} < K$. Final Answer:

$$\boxed{\frac{-40}{27} < K < 8}$$