USEFUL LIMITS

You should **know** the following limits. They may be used on Test 3 and the Final. The limits involving n are useful when using the root and ratio tests. The limits involving x are included, in case you are interested.

"$\rightarrow$" means "$\lim\limits_{n\to\infty}$"

0) $a^n \to 0$ if $|a| < 1$ — $\lim\limits_{x\to\infty} a^x = 0$ if $0 < a < 1$

1) $a^{\frac{1}{n}} \to 1$ if $a > 0$ — $\lim\limits_{x\to\infty} a^{\frac{1}{x}} = 1$ if $0 < a < 1$

2) $n^{\frac{1}{n}} \to 1$ — $\lim\limits_{x\to\infty} x^{\frac{1}{x}} = 1$

3) $\left(1 + \dfrac{a}{n}\right)^n \to e^a$ — $\lim\limits_{x\to\infty} \left(1 + \dfrac{a}{x}\right)^x = e^a$

4) $\dfrac{\ln n}{n^p} \to 0$ if $p > 0$ — $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^p} = 0$ if $p > 0$

5) $\dfrac{n^p}{b^n} \to 0$ if $b > 1$ — $\lim\limits_{x\to\infty} \dfrac{x^p}{b^x} = 0$ if $b > 1$

6) $\dfrac{a^n}{n!} \to 0$ — The continuous version involves the Gamma function.

7) $\lim\limits_{n\to\infty} \dfrac{a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0}{b_m n^m + b_{m-1} n^{m-1} + \cdots + b_1 n + b_0} = \lim\limits_{n\to\infty} \dfrac{a_k n^k}{b_m n^m}$ — The continuous version is, of course, also true.

Notes:

A) From 2) and the "pinching" or "squeeze" theorem it follows that $(\ln n)^{1/n} \to 1$ and then that $(\ln \ln n)^{1/n} \to 1$ and then ....

B) 4), 5), and 6) show the relative rates at which $\ln n \to \infty$, powers of n go to ∞, exponentials go to ∞, and factorials go to infinity.