# CURRICULUM VITAE

**Alex Jay Feingold**

**May 31, 2022**

**Department of Mathematical Sciences**
**SUNY-Binghamton, Binghamton, NY 13902-6000**

## Personal

**Born:** April 1, 1950, Baltimore, Maryland, USA

**Permanent Address:** 45 Matthews Street, Binghamton, NY 13905-4038

**Phone Numbers:** (607) 729-3637 (home), (607) 761-9850 (cellphone)

**Email:** alex@math.binghamton.edu

**Spouse:** Nancy Tittler (Married December 18, 1977)

**Children:** Emily Ruth (Born Aug. 27, 1985), Judith Marian (Born Mar. 10, 1988)

**Grandchild:** Iris Claudia Feingold Bergdall (Born Mar. 25, 2018)

## Education

1973-77 Ph.D. Yale University, New Haven, CT, Department of Mathematics

1971-73 Health Services Officer (Computer Programming), Health Services Research, U.S. Public Health Service Hospital, 3100 Wyman Park Drive, Baltimore, MD 21211

1967-71 B.A., M.A. Johns Hopkins University, Baltimore, MD, Major: Mathematics

## Dissertation

*Tensor products of modules for Lie algebras*
Advisor: Professor George B. Seligman, Yale University 1977

## Academic Honors

1987-88 Member of the Institute for Advanced Study, Princeton, NJ

1984 Member of the Mathematical Sciences Research Institute, Berkeley, California, for part of the summer.

1982-83 Member of the Institute for Advanced Study, Princeton, NJ

1973-77 Yale University Fellowship

1971 Gilman Fellowship, Johns Hopkins University

1971 Junior Instructorship, Johns Hopkins University

1969-70 Phi Beta Kappa

1967-70 U.S. Senatorial Scholarship

## Grants

(1) 1994-96 National Security Agency Grant, Mathematical Sciences Program (2 years), for "Vertex Operator Algebras and Representation Theory"

(2) 1987-88 Grant-in-Aid from the Institute for Advanced Study, Princeton, NJ

(3) 1987 National Science Foundation Grant (2 years) for "Affine and Hyperbolic Kac-Moody Algebras"

(4) 1985 National Science Foundation Grant (2 years) for "Affine and Hyperbolic Kac-Moody Algebras"

(5) 1985 National Science Foundation Grant, Mathematical Sciences Research Equipment, (1 year), jointly with other Department members

(6) 1985 Dean's Research Semester Award

(7) 1984 SUNY Faculty Research Fellowship Award

(8) 1982 SUNY Faculty Research Fellowship Award

(9) 1980-81 National Science Foundation Grant (2 years) for "Generalized Cartan Matrix Lie Algebras"

(10) 1980 SUNY Faculty Research Fellowship Award (declined because of NSF grant)

(11) 1978-79 National Science Foundation Grant for "Generalized Cartan Matrix Lie Algebras and Power Series Identities"

## Professional and Honor Societies

American Mathematical Society

Phi Beta Kappa (President of Local Chapter at Binghamton University)

Pi Mu Epsilon (Permanent Faculty Correspondent and Charter Member of the New York Alpha Omicron Chapter at Binghamton University)

## Professional Experience

1998- Professor of Mathematics, SUNY, Binghamton, NY

1987-97 Associate Professor of Mathematics, SUNY, Binghamton, NY

1987-88 Member of The Institute for Advanced Study, Princeton, NJ

1982-83 Member of The Institute for Advanced Study, Princeton, NJ

1979-87 Assistant Professor of Mathematics, SUNY, Binghamton, NY

1977-79 Visiting Assistant Professor of Mathematics, Drexel University, Philadelphia, PA

1975-77 Graduate Teaching Assistant, Mathematics, Yale University, New Haven, CT

1970-71 Junior Instructor, Mathematics, Johns Hopkins University, Baltimore, MD

## Professional Activities

Reviewer for the Mathematical Reviews since 1980 (over 56 papers reviewed).

Refereed papers for: Transactions of the American Mathematical Society, Journal of Algebra, Journal fur die Reine und Angewandte Mathematik, Journal of Number Theory, Conference Proceedings on Lie Algebras and Related Topics at Madison, Wisconsin, 1988, Duke Mathematical Journal, Journal of Pure and Applied Algebra, Advances in Mathematics, Communications in Mathematical Physics, Proceedings of the Conference *Moonshine*, *The Monster*, *and Related Topics*, 1994, Journal of Physics A, Journal of Mathematical Physics, Letters in Mathematical Physics, Pacific Journal of Mathematics.

Reviewed Manuscript of a proposed Linear Algebra textbook for John Wiley & Sons, Inc. Publishers (consulting activity).

Reviewer of grant proposals for the National Science Foundation, for the National Security Agency, and for the Croatian National Science Foundation.

Wrote a book review published in the Bulletin of the American Mathematical Society, Vol. 25, No. 2, Oct. 1991, 432-440: "Group Theory in Physics, Volume III, Super-symmetries and Infinite Dimensional Algebras", by J. F. Cornwell, Techniques of Physics: 10, ed. N. H. Marsh, Academic Press, 1989.

Served as a mentor during 2009-2010 for a high school student (Elizabeth Dwornik, Union-Endicott High School) doing a mathematics research project for the Intel Science Talent Search. This culminated in the joint writing of the expository paper "Matrix Realizations of Hyperbolic Triangle Groups".

## Invited Lectures

Cornell Algebra Seminar

Temple Univesity Colloquium

Yale University Colloquium

Yale University Lie Groups Seminar

Rutgers University Lie Algebras and Lie Groups Seminar

University of Maryland Number Theory Seminar

University of Rochester, Department of Physics

Baruch College, CUNY, Department of Mathematics

University of New Hampshire, Durham, Department of Mathematics

SUNY at Geneseo, Department of Mathematics

CUNY, Graduate Center, Department of Mathematics

Cornell Lie Groups Seminar

Various American Math Society Meetings

Mathematical Research Institute, Oberwolfach, Germany

Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India

Max Planck Institute for Gravitational Physics, Potsdam, Germany

College of Charleston, Charleston, South Carolina

Illinois State University, Normal, Illinois

University of Alberta, Edmonton, Alberta, Canada

Centre de recherches mathmatiques (CRM), Montreal, Quebec, Canada

University of Connecticut, Storrs, Connecticut

"Representation Theory XIII", Inter-University Centre, Dubrovnik, Croatia, June 20-27, 2013

International Congress of Mathematicians 2014, Satellite Conference on "Representation Theory and Related Topics", at the Exco Convention Center, Daegu, South Korea, Aug 6-9, 2014

Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 2015

Mathematics Department, University of Virginia, Charlottesville, VA, Oct 2016

**Conferences and Workshops Attended or Organized**

Attended a Joint Summer Research Conference in the Mathematical Sciences, *Conformal Field Theory, Topological Field Theory, and Quantum Groups*, Mount Holyoke College, South Hadley, MA, June 13-19, 1992.

Attended a NSF-CBMS Conference on "Applications of the Representation Theory of Quantum Affine Lie Algebras to Solvable Lattice Models", North Carolina State University, Raleigh, North Carolina, June 1-5, 1993.

Attended a Joint Summer Research Conference in the Mathematical Sciences, *Moonshine, The Monster, and Related Topics*, Mount Holyoke College, South Hadley, MA, June 18-24, 1994.

Attended the Conference on Infinite-Dimensional Lie Theory and Conformal Field Theory, University of Virginia, Charlottesville, Virginia, May 23-27, 2000.

Attended the workshop "Conformal Field Theory and Supersymmetry" April 15–19, 2002, Mathematical Sciences Research Institute, Berkeley, California.

Attended the workshop "Moonshine – the First Quarter Century and Beyond. A Workshop on the Moonshine Conjectures and Vertex Algebras" July 5–13, 2004, Heriot-Watt Univeristy, International Centre for Mathematical Sciences, Edinburgh, Scotland.

Organized the Special Session on "Lie Algebras, Conformal Field Theory, and Related Topics", at the $990^{th}$ Sectional Meeting of the American Mathematical Society, Oct. 11–12, 2004, Binghamton University, Binghamton, NY (with co-organizers Chongying Dong and Gaywalee Yamskulna).

Co-organizer of a Special Session on "Theory of Infinite-Dimensional Lie Algebras, Vertex Operator Algebras, and Related Topics", at the $1009^{th}$ Sectional Meeting of the American Mathematical Society, Oct. 8–9, 2005, Annandale-on-Hudson, NY, Bard College (with co-organizers Antun Milas and Yi-Zhi Huang).

Attended and presented an invited talk in the Special Session on Geometric and Combinatorial Methods in Representation Theory at the 1024th American Mathematical Society meeting at Davidson College, Davidson, North Carolina, March 3-4, 2007.

Visited the Albert Einstein Institute, Max Planck Institute for Gravitational Physics, in Potsdam, Germany, July 30 - August 10, 2007, for collaboration with Hermann Nicolai and Axel Kleinschmidt on Weyl groups of hyperbolic Kac-Moody algebras. Also gave a talk on "A New Perspective on the Frenkel-Zhu Fusion Rule Theorem".

Visited the Max Planck Institute for Mathematics in Bonn, Germany, August 10 - 12, for consultations with Don Zagier, on the way to Paris.

Attended part of the XXXVIIth Paris Summer Institute on Black Holes, Black Rings and Modular Forms, at the Ecole Normale Superieure, August 13 - 15, 2007.

Attended the conference "Representation Theory and Mathematical Physics" in honor of the $60^{th}$ birthday of Gregg Zuckerman, Yale University, New Haven, CT, October 24-27, 2009.

Co-organizer of a Special Session on "Kac-Moody Algebras, Vertex Algebras, and Related Topics,", at the $1072^{nd}$ Sectional Meeting of the American Mathematical Society, September 10-11, 2011, Cornell University, Ithaca, NY, (with co-organizer Antun Milas).

Attended the conference in honor of Igor Frenkel's $60^{th}$ birthday, "Perspectives in Representation Theory", May 12-17, 2012, Yale University, New Haven, Connecticut.

Attended and presented a talk at a workshop on *Infinite dimensional Lie theory: Algebra, Geometry and Combinatorics*, Centre de recherches mathmatiques (CRM), Montreal, Quebec, Canada, August 21-24, 2012, organized by Joel Kamnitzer (University of Toronto) and Michael Lau (Universit Laval).

Attended and presented a talk at "Symmetries, unification and the search for quantum gravity", A conference on the occasion of Hermann Nicolai's 60th birthday, Sept. 6 - 8, 2012, Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 20-31, 2013.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1-19, 2013.

Invited to attend and present a talk at the special session on vertex operator algebras, Kac-Moody Lie algebras and related topics, at the conference "Representation Theory XIII", Inter-University Centre, Dubrovnik, Croatia, June 20-27, 2013.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 18-31, 2014.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1-22, 2014.

Invited speaker at the International Congress of Mathematicians 2014, Satellite Conference on "Representation Theory and Related Topics", at the Exco Convention Center, Daegu, South Korea, Aug 6-9, 2014.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 18-31, 2015.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1-17, 2015.

Invited speaker at the conference on "Generalizations of Symmetric Spaces", Nahsholim Sea Resort, Israel, June 17-24, 2015.

Invited visitor at the Universitaet zu Koeln, Mathematisches Institut, Koeln, Germany, June 25-29, 2015.

Invited to present a talk in the Special Session on Representation Theory, Vertex Operator Algebras, and Related Topics at the AMS meeting at Rutgers University, New Brunswick, November 14-15, 2015.

Invited speaker at the conference on "Lie and Jordan algebras, Their Representations and Applications-VI", Bento Goncalves, Brazil, December 13-19, 2015. Unable to attend.

Presented an invited talk in the Special Session on "Algebraic structures in mathematical physics: Lie algebras, vertex algebras, quantum algebra" at the $1117^{th}$ AMS meeting at the University of Georgia, Athens, GA, March 4-6, 2016.

Invited participant in the workshop on "Automorphic Forms, Mock Modular Forms, and String Theory" at the Simons Center for Geometry and Physics, SUNY Stony Brook, August 28 - September 3, 2016.

Presented an invited talk in the Special Session on "Representation Theory and Algebraic Mathematical Physics" at the $1126^{th}$ AMS meeting at the College of Charleston, Charleston, VA, March 10-12, 2017.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 1 - June 1, 2017.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1 - July 1, 2017.

Invited to attend the conference "Modular Forms are Everywhere", Max-Planck Institute for Mathematics, Bonn, Germany, May 22-26, 2017.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 18 - June 1, 2018.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1- July 1, 2018.

Invited to attend the mini-workshop "Vertex Operators in Mathematics and Physics ", University at Albany (SUNY), April 13 - 14, 2019.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 20 - June 1, 2019.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1 - June 30, 2019.

Co-organizer of a Special Session on "Representations of Lie Algebras, Vertex Operators, and Related Topics,", at the $1151^{st}$ Fall Eastern Sectional Meeting of the American Mathematical Society, October 12-13, 2019, Binghamton University, Binghamton, NY, (with co-organizer Christopher Sadowski).

Local coordinator and organizer for the $1151^{st}$ Fall Eastern Sectional Meeting of the American Mathematical Society, October 12-13, 2019, Binghamton University, Binghamton, NY.

Invited speaker at the Special Session on "Quantum Algebra and Geometry", at the $1155^{th}$ Spring 2020 Southeastern Sectional Meeting of the American Mathematical Society, March 13-15, 2020, University of Virginia, Charlottesville, VA. Cancelled due to COVID-19.

Invited visitor at the Institut des Hautes Etudes Scientifique (IHES), Bures-sur-Yvette, France, May 18 - 30, 2020. Cancelled due to COVID-19.

Invited visitor at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, June 1 - June 30, 2020. Cancelled due to COVID-19.

Invited visitor and speaker at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, September 13 - 16, 2022, for the conference in honor of Herman Nicolai's $70^{th}$ birthday and retirement.

## Teaching Interests

Algebra (Groups, Rings, Fields, etc.), Linear Algebra, Lie Algebras and their representations, Vertex Operators, Modular Forms, Siegel Modular Forms, Conformal Field Theory, Fusion Rules.

## Summary of Research Interests

My area of special interest is the theory of Lie algebras, their representations, connections to other parts of mathematics and applications to physics. My Ph.D. dissertation [1,2] concerned the decomposition of tensor products of finite-dimensional modules for complex semisimple Lie algebras. While still a graduate student at Yale, strongly influenced by my teacher, Jim Lepowsky, I extended my research [3,4,5] into the infinite-dimensional Kac-Moody Lie algebras, independently introduced in 1968 by V.G. Kac (M.I.T.) and R.V. Moody (University of Saskatchewan). This has been an exciting and fruitful area of research because of its remarkable connections with physics (e.g., solitons, quantum field theory, string theory) and other areas of mathematics (e.g., combinatorics, group theory, modular forms, singularities, differential equations, knot theory).

During the period from 1981 to 1991 I had several collaborations [6,7,9,10,11] with Igor Frenkel (Yale University). Our first paper [6] studied hyperbolic Kac-Moody algebras, showing one such algebra to be closely connected with the theory of Siegel modular forms of genus two and with the related problem of lifting elliptic modular forms (the Saito-Kurokawa conjecture). We also gave a construction which provided closed formulas for an infinite number of root multiplicities (on levels 0, 1 and 2). Our second paper [7] studied affine Kac-Moody algebras, providing a unified approach to constructing certain representations of all the classical affine algebras. These were based on underlying associative algebras of commutation or anticommutation relations whose bosonic or fermionic representations are important in quantum field theory. Another paper [11], with J. F. X. Ries, studied representations of hyperbolic Kac-Moody algebras, constructing all irreducible highest weight standard modules

and providing closed formulas for an infinite number of weight multiplicities (on levels 0, 1 and 2). Other collaborations [9,10], also with Ries, studied the vertex operator algebras known in physics as chiral algebras. These algebras play a central role in string theory, conformal field theory, and in the Frenkel-Lepowsky-Meurman construction of the "Monster" group. Our main objectives were to obtain independent vertex and spinor constructions of chiral algebras, the isomorphism between the two viewpoints, known as a "boson-fermion correspondence", and constructions of the exceptional affine algebra $E_8^{(1)}$ based on $D_4^{(1)}$ spinor constructions and the principle of triality. Such representations of $E_8^{(1)}$ are essential in the anomaly-free heterotic superstring theory of particle physics. Some of these results were announced in [9] at the 1988 Conference on Lie Algebras and Related Topics, Madison, Wisconsin. Those results which only involve the spinor constructions are in our Contemporary Mathematics monograph ([10]). A sequel (with Ries only) was planned to give the vertex picture and the boson-fermion correspondence, but the untimely death of Ries has prevented the completion of that project up until the present.

In [13] Weiner (my first Ph.D. student) and I completed a detailed study started by Ries and myself, of the vertex operator superalgebra, modules and intertwining operators constructed from the $c = 1/2$ Virasoro modules. There are a number of constructions known [12] of vertex operator algebras and their representations, and many of these are quite difficult to give rigorously. Of particular interest are the constructions coming from Virasoro representations where $c < 1$, known in physics as the discrete series of minimal models, and those coming from representations of affine Kac-Moody Lie algebras, also known as Wess-Zumino-Witten models. These are deeply connected with braid groups and quantum groups, topics of great current interest. The structure of the intertwining operators is governed by the fusion rules which I have studied in [14,15,16,18] with various collaborators, and which were studied in the Ph.D. thesis of my student, Omar Saldarriaga. The results found in [18] brought my research back in a full circle to where I started, showing a remarkable relationship between affine fusion rules and the tensor product multiplicities I studied in my dissertation.

More recently, I returned to the study of hyperbolic Kac-Moody Lie algebras in a collaboration with Hermann Nicolai [17] where we studied subalgebras of hyperbolic algebras, and found, for example, that all the rank 2 hyperbolic algebras with symmetric Cartan matrices are subalgebras of the rank 3 hyperbolic algebra studied in [6]. In collaborations [19,20] with Hermann Nicolai and Axel Kleinschmidt, I returned to the study of Weyl groups of hyperbolic algebras. It was the key observation of [6] that the Weyl group of the rank 3 hyperbolic algebra studied there is isomorphic to $PGL(2, Z)$, an index 2 extension of the modular group, $PSL(2, Z)$. In [20] we found a generalization to all hyperbolic Weyl groups, which we showed can be realized as certain matrix groups of $2 \times 2$ matrices with entries from one of the four normed division algebras, the reals, the complex numbers, the quaternions, or the octonions. This observation could be the starting point for much deeper structural studies of all hyperbolic Kac-Moody algebras, generalizing those found in [6]. Of particular interest is the case of the hyperbolic algebra known in physics as $E_{10}$, believed to be relevant to string theory. I started research project with Terry Gannon (University of Alberta, Edmonton, Alberta, Canada) into the possibility of applying these methods with the 24-dimensional commutative non-associative Chevalley algebra in place of the octonians to understand automorphisms of the 26-dimensional even Lorentzian lattice $II_{25,1}$ and groups

of automorphisms of the Leech lattice and other lattices.

In collaborations, started during my 2009-2010 sabbatical, with Lisa Carbone (Rutgers University) and Walter Freyn (then at the University of Dortmond, Germany), we studied Tits buildings associated with hyperbolic Kac-Moody Lie groups. We have shown how to embed these twin buildings inside the light-cone of the compact form of the Lie algebra by using the structure of the family of all Cartan subalgebras inside a hyperbolic KM algebra. This paper has been published in SIGMA 16 (2020), 045, 47 pages. This is the first time I am working with the Kac-Moody groups, rather than with the Lie algebras, and the first time I have worked with the theory of buildings.

I also collaborated with Antun Milas (SUNY, Albany), on a paper investigating the representation theory behind the appearance of the Rogers-Ramanujan series in the tensor product decomposition of two level-1 modules for the twisted affine Kac-Moody Lie algebra $A_2^{(2)}$. In particular, we explained how the sum of irreducible Virasoro characters with one (positive) central charge could equal an irreducible Virasoro character with a different (negative) central charge.

I have been working on a research project with Hermann Nicolai and Axel Kleinschmidt to study the decomposition of hyperbolic Kac-Moody algebras with respect to the Nicolai-Olive principal $so(2,1)$ Lie subalgebra and various imaginary three dimensional subalgebras. Our main focus has been on understanding the action of the corresponding imaginary root groups which allows the decomposition of the hyperbolic algebra into infinite dimensional modules where the action of generators is not locally nilpotent.

I supervised the Ph.D. research of four graduate students, Quincy Loney (finished July 2012), Christopher Mauriello (finished May 2013), Diego Penta (finished May 2016) and Joshua Carey (finished May 2022). Below are descriptions of the results they obtained.

The dissertation of Quincy Loney concerns the decomposition of level-1 irreducible representations of the affine Kac-Moody algebra $D_4^{(1)}$ with respect to its subalgebra $G_2^{(1)}$ in the spinor construction. This work uses the Goddard-Kent-Olive coset Virasoro construction to find two commuting Virasoro algebras, one with central charge $1/2$ and another with central charge $7/10$, which commute with the $G_2^{(1)}$ subalgebra, and generate the space of highest weight vectors giving the branching rules. Loney's dissertation was completed and defended on July 30, 2012.

Christopher Mauriello investigated a similar branching rule problem for how the level-1 irreducible representations of the affine Kac-Moody algebra $E_6^{(1)}$ decompose with respect to its subalgebra $F_4^{(1)}$. Both of those projects involved the character theory of the relevant modules, and an important role was played by certain identities for the Roger-Ramanujan functions discovered by Ramanujan.

Diego Penta's project involved the decomposition of the rank 3 hyperbolic KM algebra, $\mathcal{F}$, with respect to its rank 2 hyperbolic "Fibonacci" subalgebra, $Fib$. Penta's dissertation was defended on May 20, 2016. An interesting aspect of Penta's project was the discovery that in the $Z$-graded decomposition of $\mathcal{F}$, with respect to $Fib$, the levels $n$ for $|n| \leq 2$ each contain exactly one irreducible $Fib$-module which is integrable but not a highest or lowest weight module, as well as an infinite direct sum of highest and lowest weight irreducible modules (with multiplicities). Those "non-standard" modules have no known character formula,

but we developed a recursive algorithm for computing weight multiplicities which points towards a possible solution to that problem. That recursive algorithm actually produces a basis for each weight space by giving an efficient list of spanning vectors and dependence relations among them. It applies as well to any Kac-Moody Lie algebra (for which there is a Kac-Peterson recursion for the multiplicities) and to any irreducible highest weight module (for which there is a Racah-Spieser recursion for the multiplicities). It may also apply to Borcherds algebras.

Joshua Carey's dissertation also studied a branching rule problem, how the level-1 irreducible representation of the affine Kac-Moody algebra $E_8^{(1)}$ decomposes with respect to its semisimple subalgebra $F_4^{(1)} \oplus G_2^{(1)}$. Since that is a conformally embedded irregular subalgebra, the coset Virasoro techniques that worked for Loney and Mauriello were not helpful. The result consists of the direct sum of just two tensor products, each a highest weight level-1 irreducible module for $F_4^{(1)}$ tensor a highest weight level-1 irreducible module for $G_2^{(1)}$. While various authors had claimed that result in the literature, there was no complete published proof until this thesis. Carey used many theta function identities, the Kac-Peterson formula for characters of irreducible highest weight modules, string functions, and the horizontal specializations of the characters. As in the dissertations of Loney and Mauriello, an important role was played by certain identities for the Roger-Ramanujan functions discovered by Ramanujan.

I completed a project with postdoctoral visitor Daniel Valliéres studying the Weyl groups of certain hyperbolic Kac-Moody Lie algebras, related to the earlier work I did with Kleinschmidt and Nicolai. This project uses matrices over a Clifford algebra to define Vahlen groups which contain certain hyperbolic Weyl groups.

While visiting IHES a few years ago, I started a research project with Prof. Robert Penner (Rene Thom Chair at the IHES), which attempts to relate the building structure found in Carbone-Feingold-Freyn for rank 3 hyperbolic KM groups, to the Ptolemy group, PPSL(2,Z), isomorphic to the Thompson group, F. A beautiful proposal by Penner to realize such a building in hyperbolic 3-space was shown not to work, but there are reasons to believe that it should work on an infinite dimensional hyperbolic space, $H^\infty$. I am discussing such a project with Walter Freyn.

**Invited Addresses**

The Special Session on Lie algebras, organized by Maria Wonenberger at the 775th Meeting of the American Mathematical Society at Bloomington, IN, April 11-12, 1980.

The Special Session on Kac-Moody Lie Theory, organized by Howard Garland and James Hurley at the 789th Meeting of the American Mathematical Society at the University of Massachusetts, Amherst, October 16-18, 1981.

The workshop on Vertex Operators in Mathematics and Physics, organized by James Lepowsky at the Mathematical Sciences Research Institute in Berkeley, CA, November 10-17, 1983.

The Lie Algebras and Related Topics Conference at the University of Wisconsin, Madison, organized by J. Marshall Osborn and Georgia Benkart, May 22-June 1, 1988.

The 1991 American Mathematical Society Summer Research Institute, on Algebraic Groups and Their Generalizations, Pennsylvania State University, University Park, PA, July 8-26, 1991.

The Special Session on Rings and Representations, organized by Martin Lorenz and Shari A. Prevost at the 868th Meeting of the American Mathematical Society, Temple University, Philadelphia, PA, October 12-13, 1991.

The Structure and Representation Theory of Lie Algebras conference in honor of George Seligman, April 10-12, 1992, Yale University, New Haven, CT.

The $884^{th}$ Meeting of the AMS, Special Session on Lie Theoretical Methods in Mathematical Physics, Sept. 18-19, 1993, Syracuse University, Syracuse, NY.

The Joint Summer Research Conference in the Mathematical Sciences, *Moonshine, The Monster, and Related Topics*, Mount Holyoke College, South Hadley, MA, June 18-24, 1994.

The $906^{th}$ Meeting of the AMS, Special Session on Quantum Kac-Moody Lie Algebras and Related Topics, Nov. 17-18, 1995, Greensboro, North Carolina.

The $922^{nd}$ Meeting of the AMS, Special Session on VOA's, Monstrous Moonshine and Related Topics, May 2-4, 1997, Detroit, Michigan.

The Conference on Generalized Kac-Moody Algebras at the Mathematical Research Institute at Oberwolfach, Germany, organized by Richard Borcherds and Peter Slodowy, July 19-25, 1998.

The $943^{rd}$ Meeting of the AMS, Special Session on Representations of Lie Algebras, April 24-25, 1999, State University of New York at Buffalo, NY.

Infinite Dimensional Lie Theory and It's Applications Program, Workshop on Vertex Operator Algebras in Mathematics and Physics, Oct. 23 - 27, 2000, The Fields Institute, Toronto, Ontario, Canada.

Ramanujan International Symposium on Kac-Moody Lie Algebras and Applications January 28–31, 2002, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India.

The $1024^{th}$ Meeting of the AMS, Special Session on Geometric and Combinatorial Methods in Representation Theory, March 3-4, 2007, Davidson College, Davidson, NC.

International Conference on Vertex Operator Algebras and Related Areas, A conference to mark the occasion of Geoffrey Mason's 60th Birthday, July 7-11, 2008, Mathematics Department, Illinois State University.

The $1048^{th}$ Sectional Meeting of the AMS, Special Session on Kac-Moody Algebras, Vertex Algebras, Quantum Groups, and Applications, North Carolina State University, April 5, 2009. Title of Talk: Hyperbolic Weyl Groups and Related Coxeter Groups.

The $1062^{nd}$ Sectional Meeting of the AMS, Special Session on Lie Algebras and, Representation Theory, Syracuse University, October 2, 2010. Title of Talk: Decomposition of a rank 2 hyperbolic Kac-Moody Lie algebra with respect to the Nicolai-Olive principal $so(1,2)$ subalgebra (joint with Elizabeth Jurisich).

The $1065^{th}$ Sectional Meeting of the AMS, Special Session on Kac-Moody Algebras, Vertex (Operator) algebras and Applications, University of Richmond, Richmond, VA, November 6, 2010. Title of Talk: Spinor construction of representations of affine Kac-Moody algebras of types $G_2^{(1)}$ and $D_4^{(3)}$ (joint with Quincy Loney).

The Canadian Mathematical Society Summer Meeting, Special Session on Lie Theory, University of Alberta, Edmonton, Alberta, Canada, June 3, 2011. Title of Talk: Decomposition of level-1 representations of $D_4^{(1)}$ with respect to its subalgebra $G_2^{(1)}$ in the spinor construction (joint with Quincy Loney).

A workshop on *Infinite dimensional Lie theory: Algebra, Geometry and Combinatorics*, Centre de recherches mathmatiques (CRM), Montreal, Quebec, Canada, August 21-24, 2012, organized by Joel Kamnitzer (University of Toronto) and Michael Lau (Universit Laval).

"Symmetries, unification and the search for quantum gravity", A conference on the occasion of Hermann Nicolai's 60th birthday, Sept. 6 - 8, 2012, Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany.

"Representation Theory XIII", A conference at the Inter-University Centre, Dubrovnik, Croatia, June 20-27, 2013, special session on vertex operator algebras, Kac-Moody Lie algebras and related topics.

International Congress of Mathematicians 2014, Satellite Conference on "Representation Theory and Related Topics", at the Exco Convention Center, Daegu, South Korea, Aug 6-9, 2014.

Conference on "Generalizations of Symmetric Spaces", Nahsholim Sea Resort, Israel, June 17-24, 2015.

Special Session on Representation Theory, Vertex Operator Algebras, and Related Topics, AMS meeting at Rutgers University, New Brunswick, November 14-15, 2015.

Conference on "Lie and Jordan algebras, Their Representations and Applications-VI", Bento Goncalves, Brazil, December 13-19, 2015. Unable to attend.

Special Session on "Algebraic structures in mathematical physics: Lie algebras, vertex algebras, quantum algebra" at the $1117^{th}$ AMS meeting at the University of Georgia, Athens, GA, March 4-6, 2016.

Special Session on "Representation Theory and Algebraic Mathematical Physics" at the $1126^{th}$ AMS meeting at the College of Charleston, Charleston, VA, March 10-12, 2017.

Special Session on "Quantum Algebra and Geometry", at the $1155^{th}$ Spring 2020 Southeastern Sectional Meeting of the American Mathematical Society, March 13-15, 2020, University of Virginia, Charlottesville, VA. Cancelled due to COVID-19.

Conference in honor of Herman Nicolai's $70^{th}$ birthday and retirement at the Albert Einstein Institute, Max-Planck Institute for Gravitational Physics, Potsdam, Germany, September 13 - 16, 2022.

## Publications

1. "Zones of uniform decomposition in tensor products", Proceedings of the American Mathematical Society, Vol. 70, No. 2, July 1978, 109-113.

2. "Tensor products of finite dimensional modules for complex semisimple Lie algebras", Lie Theories and Their Applications, Proceedings of the 1977 Annual Seminar of the Canadian Mathematical Congress, Queen's Papers in Pure and Applied Mathematics No. 48, Editors: A. J. Coleman and P. Ribenboim, Queen's University, Kingston, Ontario, 1978, 394-397.

3. "The Weyl-Kac character formula and power series identities", Advances in Mathematics 29, No. 3, September 1978, 271-309 (with J. Lepowsky).

4. "A hyperbolic GCM Lie algebra and the Fibonacci numbers", Proceedings of the American Mathematical Society, Vol. 80, No. 3, November 1980, 379-385.

5. "Tensor products of certain modules for the Generalized Cartan Matrix Lie Algebra $A_1^{(1)}$", Communications in Algebra, Vol. 9, No. 12, 1981, 1323-1341.

6. "A hyperbolic Kac-Moody algebra and the theory of Siegel modular forms of genus 2", Mathematische Annalen 263, 1983, 87-144 (with I. Frenkel).

7. "Classical affine algebras", Advances in Mathematics, Vol. 56, No. 2, May 1985, 117-172 (with I. Frenkel).

8. "Some applications of vertex operators to Kac-Moody algebras", Vertex Operators in Mathematics and Physics. Proceedings of a conference Nov. 10-17, 1983. Edited by J. Lepowsky, S. Mandelstam, I. M. Singer. Publications of the Mathematical Sciences Research Institute #3, Springer-Verlag, 1985, 185-206.

9. "The exceptional affine algebra $E_8^{(1)}$, triality and chiral algebras", Lie Algebras and Related Topics, Proceedings of a Conference held in Madison, Wisconsin, May 22 to June 1, 1988; Editors: G. Benkart and J. M. Osborn; Contemporary Mathematics, Vol. 110, American Mathematical Society, Providence, RI, 1989, (with Igor Frenkel and John F. X. Ries).

10. "Spinor Construction of Vertex Operator Algebras, Triality and $E_8^{(1)}$", Contemporary Mathematics, Vol. 121, American Mathematical Society, Providence, RI, 1991, 146 pp. monograph (with Igor Frenkel and John F. X. Ries).

11. "Representations of hyperbolic Kac-Moody algebras", Journal of Algebra, Vol. 156, No. 2, April 1993, 433-453 (with Igor Frenkel and John F. X. Ries).

12. "Constructions of vertex operator algebras", Proceedings of Symposia in Pure Mathematics, Vol. 56, Algebraic Groups and Their Generalizations, William J. Haboush and Brian J. Parshall, Editors, American Mathematical Society, Providence, RI, Part 2, 317-336, April 1994.

13. "Spinor construction of the $c = \frac{1}{2}$ minimal model", *Moonshine, The Monster, and Related Topics*, Contemporary Mathematics, Vol. 193, Chongying Dong and Geoffrey Mason, editors, American Mathematical Society, Providence, RI, 1995 (with John F. X. Ries and Michael Weiner), 45-92.

14. "Minimal model fusion rules from 2-groups", Letters in Mathematical Physics, Vol. 40, No. 2 (1997), 159-169, (with Füsun Akman and Michael Weiner).

15. "Type A Fusion Rules From Elementary Group Theory", Comtemporary Mathematics, Vol. 297, Proceedings of the Conference on Infinite-Dimensional Lie Theory and Conformal Field Theory, Charlottesville, VA, American Mathematical Society, Providence,RI, 2002 (with Michael Weiner), 97–115.

16. "Fusion Rules for Affine Kac-Moody Algebras", Kac-Moody Lie Algebras and Related Topics, Ramanujan International Symposium on Kac-Moody Algebras and Applications, Jan. 28-31, 2002, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India, N. Sthanumoorthy, Kailash Misra, Editors, Contemporary Mathematics, Vol. 343, American Mathematical Society, Providence, RI, 2004, 53–96.

17. "Subalgebras of hyperbolic Kac-Moody Algebras", Kac-Moody Lie Algebras and Related Topics, Ramanujan International Symposium on Kac-Moody Algebras and Applications, Jan. 28-31, 2002, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai, India, N. Sthanumoorthy, Kailash Misra, Editors, Contemporary Mathematics, Vol. 343, American Mathematical Society, Providence, RI, 2004, 97–114, (with Hermann Nicolai).

18. "A new perspective on the Frenkel-Zhu fusion rule theorem", Journal of Algebra 320 (2008), 2079–2100, (with Stefan Fredenhagen).

19. "Hyperbolic Weyl groups and the four normed division algebras", in Vertex Operator Algebras and Related Areas, Contemporary Mathematics, Vol. 497, Amer. Math. Soc., Providence, RI, (2009), 53-64 (with Hermann Nicolai and Axel Kleinschmidt).

20. "Hyperbolic Weyl groups and the four normed division algebras", Journal of Algebra 322 (2009), 1295-1339 (with Hermann Nicolai and Axel Kleinschmidt). Corrigendum to "Hyperbolic Weyl Groups and the Four Normed Division Algebras" [J. Algebra 322 (2009) 1295-1339], J. Algebra 489 (2017), 586-587. The arXiv copy of the 2009 paper has been updated to contain the corrections. Another paper with the same title and authors as the 2009 paper was published in the proceedings of an international

conference on vertex operators and related areas in honor of Geoffrey Mason. That conference took place at Illinois State University in July, 2008. That shorter paper is an introduction to and announcement of the results in this paper, and appeared in Vertex operator algebras and related areas, Contemporary Mathematics, vol. 497, Amer. Math. Soc., Providence, RI (2009), 53-64.

21. "Matrix realizations of hyperbolic triangle groups", completed July 2010, (with Elizabeth Dwornik), unpublished.

22. "The 3-State Potts model and Rogers-Ramanujan series", Central European Journal of Mathematics, 11(1), 2013, pp. 1–16 (with Antun Milas).

23. "Weyl groups of some hyperbolic Kac-Moody algebras", Journal of Algebra 500 (2018), 457-497 (with Daniel Vallières). DOI information: 10.1016/j.jalgebra.2017.05.003

24. "A Lightcone embedding of the twin building of a hyperbolic Kac-Moody group", (with Lisa Carbone and Walter Freyn), SIGMA 16 (2020), 045, 47 pages. arXiv.org/abs/1606.05638.

**Work in Progress**

1. "Structure of Cartan subalgebras in hyperbolic Kac-Moody algebras", (with Walter Freyn).

2. "Further research on hyperbolic Kac-Moody algebras", (with Hermann Nicolai and Axel Kleinschmidt).

3. "Vertex Operator Algebras, Triality and $E_8^{(1)}$", book, (with the late John F. X. Ries).

**Graduate Students**

1. Michael D. Weiner, Ph.D. 1994, Thesis: "Bosonic Construction of Vertex Operator Para-Algebras from Symplectic Affine Kac-Moody Algebras", published: Memoirs of the American Mathematical Society, Vol. 135, 1998. (Currently Associate Professor at the Altoona Campus of Penn State University.)

2. Omar Saldarriaga, Ph.D. 2004, Thesis: Fusion Algebras, Symmetric Polynomials, Orbits of $N$-Groups, and Rank-Level Duality. Published as an article in the Journal of Algebra 312 (2007), 257–293. (Currently Professor at the Universidad de Antioquia, Medelln, Colombia, South America.)

3. Quincy Loney, Ph.D. July 2012, Thesis: Decomposition of level-1 representations of $D_4^{(1)}$ with respect to its subalgebra $G_2^{(1)}$ in the spinor construction.

4. Christopher Mauriello, Ph.D. May 2013, Thesis: Branching rule decomposition of irreducible level-1 $E_6^{(1)}$-modules with respect to the affine subalgebra $F_4^{(1)}$.

5. Diego Penta, Ph.D. May 2016: Thesis: Decomposition of the rank 3 hyperbolic Kac-Moody Lie algebra, $\mathcal{F}$, with respect to its rank 2 hyperbolic subalgebra, $\mathcal{F}ib$.

6. Joshua Carey, Ph.D. May 2022: Thesis: Branching rule decomposition of the irreducible level-1 $E_8^{(1)}$-module with respect to the irregular subalgebra $F_4^{(1)} \oplus G_2^{(1)}$.