MATH. 341. Test No. 2. October 20, 2000.

Name: ....

1. (10 points) A marketing survey indicates that 60 % of the population owns an automobile, 30 % owns a house and 70 % owns either an automobile or a house. A person owning an automobile is chosen at random, what is he probability that this person owns a house?

Answer:

Let $A = \{\text{person owns an automobile}\}$ and let $B = \{\text{person owns a house}\}$. We need to found $P(B|A)$.

By the inclusion–exclusion formula

$$.70 = P(A \cup B) = P(A) + P(B) - P(A \cap B) = .6 + .3 - P(A \cap B)$$

we get $P(A \cap B) = 0.2$.

So, $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.2}{.6} = .3333$.

2. (20 points) Three chips are selected at random from a bowl containing 5 white, 4 black, and 7 red chips.

(a) Find the probability that these three chips are alike in color when the chips are selected without replacement.

(b) Find the probability that these three chips are alike in color when the chips are selected with replacement.

Answer:

To get 3 chips are alike in color means to get either 3 white chips, or 3 black chips or 3 red chips.

(a) When we sample without replacement, since there are 16 balls, we have $\binom{16}{3}$ choices to choose 3 balls. To get 3 white balls, we have $\binom{5}{3}$ choices. To get 3 black balls, we have $\binom{4}{3}$ choices. To get 3 red balls, we have $\binom{7}{3}$ choices. So, the probability that the 3 chosen balls are alike is

$$P(3 \text{ white chips }) + P(3 \text{ black chips }) + P(3 \text{ red chips }) = \frac{\binom{5}{3}}{\binom{16}{3}} + \frac{\binom{4}{3}}{\binom{16}{3}} + \frac{\binom{7}{3}}{\binom{16}{3}} = 0.0875.$$

(b) When we sample without replacement, we always have the same number of the balls. Hence, the probability of obtaining a ball of a determined color is always the same. In particular, the probability of getting a white ball is always $\frac{5}{16}$; the probability of getting a black ball is always $\frac{4}{16}$; and the probability of getting a red ball is always $\frac{7}{16}$. So,

$$P(3 \text{ white chips }) + P(3 \text{ black chips }) + P(3 \text{ red chips }) = \left(\frac{5}{16}\right)^3 + \left(\frac{4}{16}\right)^3 + \left(\frac{7}{16}\right)^3 = 0.129883.$$

3. (20 points) One of the questions asked by an insurer on an application to purchase a life insurance policy is whether or not the applicant is a smoker. The insurer knows that the proportion of smokers in the general population is 30 %, and assumes that this represents the proportion of applicants who are smokers. The insurer has also obtained information regarding the honesty of applicants: 40 % of applicants that are smokers say that they are non–smokers on their applications, none of the applicants who are non–smokers lie on their applications. What proportion of applicants who say they are non–smokers are actually non-smokers?

Answer:

We apply the Bayes' formula. Let $A = \{\text{person says he/she does not smoke}\}$, let $B_1 = \{\text{person smokes}\}$ and let $B_2 = \{\text{person does not smoke}\}$. We need to found

$$P(B_2|A) = \frac{P(B_2)P(A|B_2)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)} = \frac{.7 \cdot 1}{(.7 \cdot 1) + (.3 \cdot .4)} = \frac{.7}{.82} = .5833.$$

4. (15 points) Three missiles, whose probabilities of hitting a target are 0.7, 0.8 and 0.9, respectively, are fired at a target. Assuming independence, what is the probability that the target is hit?

Answer:

Let $A = \{\text{1st missil hits the target}\}$, let $B = \{\text{2nd missil hits the target}\}$ and let $C = \{\text{3rd missil hits the target}\}$. We have that

$$P(A \cup B \cup C) = 1 - P(A^c \cap B^c \cap C^c) = 1 - (.3 \cdot .2 \cdot .1) = .994.$$

5. (20 points) Suppose that the chance of triples in human births is .0001. What is the probability of observing at least two sets of triples in 10,000 human births?

Answer:

In this problem we can use the Poisson approximation to the binomial distribution with $\lambda = np = 10,000 \cdot .0001 = 1$. We have that

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-1} - e^{-1} = 0.2642.$$

6. (15 points) Customers at Fred's Cafe win a 100 dollar prize if their cash register receipts show a star on each of the five consecutive days Monday, ...,Friday in any one week. The cash register is programmed to print starts on a randomly selected 25 % of the receipts. If Mark eats at Fred's once each day for four consecutive weeks and the appearance of stars is an independent process, what is the expected number of dollars won by Mark in the four-week period?

Answer:

Let X_1, X_2, X_3, X_4 be the Mark's winings in each of the four weeks. For each $i = 1, 2, 3, 4$, we have $P(X_i = 100) = .25^5 = \frac{1}{1024}$ and $P(X_i = 0) = 0$. So, $E[X_i] = 100 \cdot \frac{1}{1024}$. The expected number of dollars won by Mark in the four-week period is

$$E[X_1 + X_2 + X_3 + X_4] = 4 \cdot 100 \cdot \frac{1}{1024} = \$0.390625 \text{ or } 39 \text{ cents.}$$