MATH. 341. Test No. 3 /Special November 22, 1999.

We use the tables in the web page http://www.math.binghamton.edu/arcones/341/table.html and interpolation

Answer all Questions.

1. (20 points) A random variable X has a binomial distribution with parameters $n = 1000$ and $p = 0.25$. Approximate the probability of the event $\{X < 253\}$.

Answer: Since X is a binomial random variable

$$E[X] = np = 1000 \cdot (.25) = 250$$

and

$$\mathrm{Var}(X) = np(1-p) = 1000 \cdot (.25) \cdot .75 = 187.5$$

Using the central limit theorem with continuity correction,

$$\Pr\{X < 253\} = \Pr\{X \le 252.5\} \approx \Pr\{N(0,1) \le \frac{252.5 - 250}{\sqrt{187.5}}\} = \Phi(.1826)$$

$$= (1 - .826)\Phi(.1) + .826\Phi(.2) = (1 - .826) \cdot .539828 + .826 \cdot .579260 = .5724.$$

2. (20 points) A bin contains $N = 300$ printed circuits, of which $M = 35$ are defective PC's. If $n = 150$ PC's are selected at random without replacement (RSWOR), what are the expected number and standard deviation of the number of defective PC's, X, in the sample?

Solution: Since, we are sampling without replacement, we have a hypergeometric distribution. The mean and variance of this distribution are in page 107 of the textbook:

$$E[X] = \frac{n \cdot M}{N} = \frac{150 \cdot 35}{300} = 17.5$$

and

$$\mathrm{Var}(X) = \frac{n \cdot M}{N} \cdot \left(1 - \frac{M}{N}\right) \cdot \left(1 - \frac{n-1}{N-1}\right) = \frac{150 \cdot 35}{300} \cdot \left(1 - \frac{35}{300}\right)\left(1 - \frac{149}{299}\right)$$

$$= \frac{150 \cdot 35 \cdot 265 \cdot 150}{300 \cdot 300 \cdot 299} = 7.7550$$

The standard deviation is STD =2.7848.

3. (20 points) The length of a steel rod has a normal distribution N(20,5). Find the probability $P\{13 < X < 27\}$.

Answer:

$$\Pr\{13 < X < 27\} = \Pr\left\{\frac{13-20}{5} < N(0,1) < \frac{27-20}{5}\right\}$$

$$= \Pr\{-1.4 < N(0,1) < 1.4\} = \Phi(1.4) - \Phi(-1.4) = 2 \cdot \Phi(1.4) - 1 = 2 \cdot .919243 - 1 = .838486.$$

4. (20 points) A concrete cube has a compressive-strength X which has a log-normal distribution LN(μ, σ) with parameters $\mu = 4$ and $\sigma = 2$.

a.) What is the probability that a concrete cube will have compressive-strength greater than 300 (pounds/inch^2)?

b.) What is the probability in a random sample of $n = 10$ concrete cubes, there will be more than 3 cubes with compressive-strength greater than 300 (pounds/inch2)?

Solution:

(a) We transform the lognormal to a normal distribution and use minitab to find probabilities:

$$\Pr\{X > 300\} = \Pr\{\ln(X) > \ln 300\} = \Pr\left\{N(0,1) > \frac{\ln(300) - 4}{2}\right\}$$

$$= \Pr\{N(0,1) > .8529\} = 1 - \Phi(.8529) = 1 - ((1 - .529)\Phi(.8) + .529\Phi(.9))$$

$$= 1 - ((1 - .529) \cdot .788145 + .529 \cdot .815940) = 1 - .802849 = .197151.$$

(b) If Y is the number of cubes with compressive-strength greater than 300, then Y has binomial distribution with $n = 10$ and $p = .1969$. So,

$$\Pr\{Y \geq 4\} = 1 - \Pr\{Y = 0\} - \Pr\{Y = 1\} - \Pr\{Y = 2\} - \Pr\{Y = 3\}$$

$$= 1 - .8031^{10} - \binom{10}{1}.1969^1 \cdot .8031^9 - \binom{10}{2}.1969^2 \cdot .8031^8 - \binom{10}{3}.1969^3 \cdot .8031^7 = .11548$$

5. (20 points) A random variable X has a rectangular distribution $R(-5, 5)$. Find the expected value and the variance of X.

Rectangular is a nickname for the uniform distribution. We have that $E[X] = \frac{a+b}{2} = 0$ and $\mathrm{Var}(X) = \frac{(b-a)^2}{12} = \frac{10^2}{12} = 8.3333333333$.