Math 447. Introduction to Probability and Statistics I. Fall 1998.

Schedule: M. W. F.: 08:00-09:30 am. SW–323

Textbook: Introduction to Mathematical Statistics by R. V. Hogg and A. T. Craig, 1995, Fifth edition. Prentice Hall.

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Office hours: M: 9:40–10:30; W: 13:00–14:00; Fr: 9:40–10:30. Or by appointment. Feel free to come to my office at any time.

Grading: Two midterms (60%), and a final (40%). If you miss an exam, your score for that exam will be a zero.

Midterms exams: Friday, October 2; Wednesday, November 4 (in the classroom).

Final exam: Sometime in Dec 14–18.

Course description: This course is a half of a series covering the whole book. The first chapters are a probabilistic preparation for the rest of the chapters devoted to Mathematical Statistics. We will go from Chapter 1, until time allows, likely until the end of Chapter 5.

Homework: You do not have to hand the homework. I will do some of these problems in class. I will provide you with the solutions to these problems. The problems assigned as homework are:

<ins>Chapter 1.</ins> 1, 2, 4, 7, 15, 18, 20, 22, 24, 25, 26, 27, 28, 30, 31, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 52, 53, 54, 55, 57, 61, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 83, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 105, 106, 107, 108, 109, 113, 114, 115, 116, 120, 121, 124, 129, 131, 132, 133, 134.

<ins>Chapter 2.</ins> 1, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 16, 19, 20, 21, 22, 24, 25, 26, 28, 29, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 49, 52.

<ins>Chapter 3.</ins> 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 15, 16, 17, 20, 21, 23, 24, 26, 28, 30, 31, 34, 35, 36, 37, 38, 39, 40, 46, 48, 49, 50, 51, 53, 55, 58, 59, 60, 63, 70, 71, 75, 76, 79, 82, 83, 84, 85, 86, 89, 93, 95, 99.

<ins>Chapter 4.</ins> 1, 4, 6, 9, 11, 14, 15, 17, 18, 19, 20, 21, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 70, 71, 74, 76, 77, 78, 79, 80, 81, 82, 84, 86, 87, 88, 89, 90, 92, 94, 95, 96, 97, 100, 101, 102, 103, 104, 106, 108, 110, 111, 113, 114, 115, 116, 117, 118, 122, 123, 126, 127, 128, 129, 130, 135, 138, 141, 144, 146, 148, 150, 151, 152, 154, 158, 159.

<ins>Chapter 5.</ins> 1, 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 21, 22, 23, 24, 27, 28, 29, 32, 33, 34, 36, 37, 39, 40, 45, 46, 47, 48, 50, 51.

Lective days:

	Aug	September					October				November				Dec
M	31		14		28	5	12	19	26	2	9	16	23	30	7
W	2	9	16	23		7	14	21	28	4	11	18	25	2	9
F	4	11	18	25	2	9	16	23	30	6	13	20		4	11

Math 447. First Midterm. Friday, October 2, 1998.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. In testing the water supply for various cities for two kinds of impurities commonly found in water, it was found that 20 % of the water supplies had neither sort of impurity, 40 % had an impurity of type A, and 50 % had an impurity of type B. If a city is chosen at random, what is the probability that is water supply has exactly one type of impurity?

2. Three chips are selected at random and without replacement from a bowl containing 5 white, 4 black, and 7 red chips. Find the probability that these three chips are alike in color.

3. If it is assumed that all the $\binom{52}{5}$ poker hands are equally likely, what is the probability of getting 2 or more aces in a poker hand?

4. On a six–question multiple choice test there are five possible answers of which one is correct and four incorrect. If a student guesses randomly and independently find the probability that he gets at least 2 questions right?

5. Factories A, B, and C produce respectively 20, 30 and 50 % of a certain company's output. The items produced in A, B, and C are 1, 2, and 3 percent defective, respectively. We observe one item from the company's output at random and find it defective. What is the conditional probability that the item was from A?

6. The game of bridge is played by four players: north, south, east and west. Each of these players receive 13 cards. In a game of bridge West has exactly 4 spades, what is the probability of East having exactly 3 spades?

7. If the cumulative distribution function of the random variable X is given by

$$F(b) = \begin{cases} 0 & \text{if } b < -2 \\ 1/4 & \text{if } -2 \le b < -1 \\ 3/5 & \text{if } -1 \le b < 1 \\ 7/9 & \text{if } 1 \le b < 2 \\ 1 & \text{if } 2 \le b. \end{cases}$$

Find the probability density function p_Y of the random variable $Y = X^2$.

8. Let X be a discrete random variable such that $P(X = x) > 0$, if $x = 1, 2, 3$ or 4, $P(X = x) = 0$, otherwise. Suppose that the distribution function of X is $\frac{x(x+1)}{20}$ at the values $x = 1, 2, 3, 4$. Find the expected value of the random variable $Y = 2X - 1$.

9. Let X be a random variable with probability density function

$$f(x) = \begin{cases} x + \frac{1}{2} & \text{if } 0 < x < 1, \\ 0 & \text{else.} \end{cases}$$

Find the probability density function of the random variable $Y = \ln X$.

10. The measured radius a circle R has probability density function $f(r) = 6r(1-r)$, $0 < r < 1$, zero elsewhere. Find the expected area.

Math 447. Second Midterm. Wednesday, November 4, 1998.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. If the cumulative distribution function of the random variable X is given by

$$F(b) = \begin{cases} 0 & \text{if } b < -4 \\ 1/8 & \text{if } -4 \le b < -3 \\ 3/8 & \text{if } -3 \le b < 2 \\ 3/4 & \text{if } 2 \le b < 5 \\ 1 & \text{if } 5 \le b. \end{cases}$$

Find the mean and the variance of X.

2. Let $f(k) = \Pr\{X = k\} = p(1-p)^k$, $k = 0, 1, 2, ...$, where $0 < p < 1$, be the pdf of the r.v X. Find the mgf and the mean of the r.v. X.

3. A television store owner figures that 45 percent of the customers entering his store will purchase an ordinary television set, 15 percent will purchase a color television set, and 40 percent will just be browsing. If 5 customers enter his store on a given day, what is the probability that he will sell exactly 2 ordinary sets and 1 color set on that day?

4. A man and a woman agree to meet at a certain location about 12:30 p.m. If the man arrives at a time uniformly distributed between 12:15 and 12:45 and the woman independently arrives at a time uniformly distributed between 12:30 and 1 pm, find the probability that the man arrives first?

5. Let X and Y have the pdf

$$f(x, y) = \begin{cases} 3y & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{else} \end{cases}$$

Find the pfd of $Z = Y - X$.

6. Let X and Y have the pdf

$$f(x, y) = \begin{cases} cx & \text{if } x - 1 \le y \le 1 - x,\ 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

Find c, the mean and the variance of X and Y, and the covariance of X and Y.

7. Let X and Y have the pdf

$$f(x, y) = \begin{cases} 6(y - x) & \text{if } 0 \le x \le y \le 1 \\ 0 & \text{else} \end{cases}$$

Find the marginal pfd's, the conditional pdf's and $E[X|Y = y]$ and $E[Y|X = x]$.

8. A box contains 6 red, 8 green and 10 blue balls. Six balls are drawn successively without replacement. What is the probability that there are exactly 3 red balls, 2 green balls and one blue ball in the six withdrawn balls?

9. Find the probability that the fifth head is observed on the tenth independent flip of an unbiased coin.

10. A certain type of aluminum screen that is 2 feet wide has on the average one flaw in a 100–foot roll. Find the probability that a 50–foot roll has no flaws.

Math 447. Final Exam. Wednesday, December 16, 1998.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. Bowl I contains 7 red and 3 white chips and bowl II has 4 red and 6 white chips. One chip is selected at random from I and transferred to II. Three chips are then selected at random and without replacement from II. What is the probability that all three are white?

2. Find the mean and the variance of the random variable X with distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{4} & \text{if } 0 \le x < 1 \\ \frac{x^2}{4} & \text{if } 1 \le x < 2 \\ 1 & \text{if } 2 \le x. \end{cases}$$

3. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = cx^9 e^{-x}, \quad x > 0.$$

Compute c and the expected lifetime of such a tube.

4. Let X and Y have the joint pdf $f(x, y) = 6x$, $0 < x < y < 1$, zero elsewhere. Find the pdf of $Z = Y - X$.

5. Suppose that X is a gamma random variable with mean 1 and variance 2. Find the density of the random variable Y defined by $Y = 3X + 6$.

6. Let X_1 and X_2 be independent random variables with joint pdf

$$f(x_1, x_2) = \frac{x_1(4 - x_2)}{36}, x_1 = 1, 2, 3, \; x_2 = 1, 2, 3,$$

and zero elsewhere. Find the pdf of $Y = X_1 - X_2$.

7. Let X_1 and X_2 be two independent r.v.'s with uniform distribution in the interval $(0, 1)$. Find the joint density of $Y_1 = X_1 X_2$ and $Y_2 = \frac{X_1}{X_2}$. Graph the region in the (y_1, y_2) plane where the pdf of (Y_1, Y_2) is positive.

8. Let X_1 and X_2 be 2 random variables. Suppose that each $E[X_1] = -1$, $E[X_2] = 2$, $\mathrm{Var}(X_1) = 2$, $\mathrm{Var}(X_2) = 8$ and the correlation coefficient of X_1 and X_2 is $-1/2$. Find the mean and the variance of $Y = 3 + X_1 - 2X_2$.

9. Let S^2 be the variance of a random sample of size 10 from the normal distribution $N(10, 40)$. Find $\Pr\{10.8 \le S^2 \le 76\}$.

10. Let $\bar{x}$ be the mean of a random sample of size n from a normal distribution with mean 100 and variance $\sigma^2 = 4$. Find the smallest n so that $\Pr\{99.9 \le \bar{x} \le 100.1\} \ge .95$.

11. Suppose that in 4–child families, each child is equally likely to be a boy or a girl, independently of the others. Which would then be more common, 4–child families with 2 boys and 2 girls, or 4–child families with different numbers of boys and girls?

12. A student played the board game Pinocchio with his friend's nephews. In this game, to try to escape the great white whale, a player rolls a fair six–sided die 20 times, adding together these 20 outcomes. To escape the whale, the player must receive a score of at least 75. For a score of less than 75 the player is "eaten by the whale". Find, approximately, the probability of escaping the whale.