Math 447. 1st Homework. First part of Chapter 2. Due Friday, September 17, 1999.

1. How many different seven–place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

2. A shipment of 50 mechanical devices consists of 42 good ones and eight defective. An inspector selects five devices at random without replacement. What is the probability that exactly three are good?

3. Among the 20 candidates for four positions on a city council, six are Democrats, ten are Republicans and four are independents. In how many ways can the four councilmen be chosen so that two are Democrats and two are Republicans?

4. Three chips are selected at random and without replacement from a bowl containing 5 white, 4 black, and 7 red chips. Find the probability that these three chips are alike in color.

5. A woman has 10 friends, consisting of 3 couples and 4 singles. She is going to invite 5 persons to a party. How many choices does she have if every couple only will attend together?

Math 447. 2nd Homework. Second part of Chapter 2. Due Wednesday, September 29, 1999.

1. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes. What percentage of males smoke neither cigars nor cigarettes?

2. Suppose that E and F are independent events with $P(E) = P(F)$ and $P(E \cup F) = 3/4$. What is $P(E)$?

3. Three missiles, whose probabilities of hitting a target are 0.7, 0.8 and 0.9, respectively, are fired at a target. Assuming independence, what is the probability that the target is hit?

4. A history teacher tells his class that 80 percent of the students who regularly does his homework will get a passing grade, and 75 percent of the students who does not regularly do his homework will fail. Also, he figures that 60 percent of his students regularly do their homework. What is the probability that a student who gets a passing grade did his homework?

5. In a bolt factory, machines 1, 2 and 3 respectively produce 20 %, 30 % and 50 % of the total output. Of their respective outputs, 5 %, 3 % and 2 % are defective. A bolt is selected at random. Given that it is defective, what is the probability that it was made by machine 1?

Math 447. 3rd Homework. First part of Chapter 3. Due Friday, October 8, 1999.

1. A player throws a fair die and simultaneously flips a fair coin. If the coin lands heads, then she wins twice the value that appears on the die, and if tails, then she wins one–half of the value that appears on the die. Determine her expected winnings.

2. The probability that Ms. Brown will sell a piece of property at a profit of \$3,000 is $\frac{3}{20}$, the probability that she will sell at a profit of \$1,500 is $\frac{7}{20}$, the probability that she will break even is $\frac{7}{20}$, and the probability that she will lose \$1,500 is $\frac{3}{20}$. What is her expected profit?

3. A fair die is rolled 20 times. Calculate the expected number of sixes in the 20 rolls.

4. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student would get 4 or more correct answers just by guessing?

5. A box contains 6 white balls and 8 black balls. Seven balls are drawn succesively with replacement. What is the probability that there are exactly 4 white balls in the 7 withdrawn balls? Repeat the problem, assuming that the withdrawn balls are not replaced.

Math 447. 4–th Homework. Second part of Chapter 3. Due Monday, October 18, 1999.

1. A certain basketball player hits a shot with probability 0.3. What is the probability that he needs to throw 10 or more shots in order to get one shot?

2. Suppose that an ordinary six–sided fair die is rolled repeatedly, and the outcome is noted in each roll. What is the probability that the third 6 occurs on the seventh roll?

3. A shipment of 50 mechanical devices consists of 42 good ones and eight defective. An inspector selects five devices at random without replacement. What is the probability that exactly three are good?

4. Suppose that the number of typographical errors on a single page of this book has a Poisson distribution with parameter $\lambda = 1/2$. Find the probability that in the first 3 pages of the book there are exactly 2 errors.

5. If Y has a geometric distribution with probability of success p, show that the moment generating function for Y is $m(t) = \frac{pe^t}{1-(1-p)e^t}$. Differentiate the moment generating function $m(t)$ to find $E[Y]$ and $V(Y)$.

Math 447. 5–th Homework. Second part of Chapter 4. Due Wednesday, October 28, 1999.

1. If the cumulative distribution function of the discrete random variable X is given by

$$F(b) = \begin{cases} 0 & \text{if } b < -2 \\ 1/4 & \text{if } -2 \le b < 1 \\ 3/5 & \text{if } 1 \le b < 7/4 \\ 7/9 & \text{if } 7/4 \le b < 3 \\ 1 & \text{if } 3 \le b. \end{cases}$$

Find the probability function of the random variable X. Find $P\{1 \le X < 3\}$.

2. Let X be a continuous random variable with density

$$f(x) = \begin{cases} c(1+x) & \text{if } 0 < x < 2, \\ 0 & \text{else.} \end{cases}$$

Find the value of c. Find the distribution function of the random variable X. Compute the expectation and the variance of the random variable X.

3. If the cumulative distribution function of the random variable Y is given by

$$F(y) = \begin{cases} 0 & \text{if } y < -1 \\ \frac{y+1}{4} & \text{if } -1 \le y < 0 \\ \frac{3y^2+4}{16} & \text{if } 0 \le y < 2 \\ 1 & \text{if } 2 \le y. \end{cases}$$

Check that F is a continuous function. Find the density function of Y. Find the mean and the variance of Y.

4. Let X be a random variable with uniform distribution on the interval $[0, 10]$. Find $P\{2 \le X \le 5 \,|\, X \le 3\}$.

5. A candymaker produces mints that have a label weight of 20.4 grams. Let X denote the weight of a single mint selected at random from the production line. Assume that X has a normal distribution with mean 21.37 and variance 0.16. Find $P(X > 22.07)$.

Math 447. 6–th Homework. Second part of Chapter 4. Due Friday, November 5, 1999.

1. The length of the time required to complete a college achievement test is found to be normally distributed with mean 70 minutes and standard deviation 12 minutes. When should the test be terminated if we wish to allow sufficient time of 90 % of the students to complete the test?

2. Let Y have density function

$$f(y) = \begin{cases} cye^{-2y} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

Find the value of c. Compute the expectation, the variance and the moment generating function of the random variable Y. Find an expression for $\mu'_k = E[Y^k]$.

3. If the annual proportion of erroneous income tax returns filed with IRS can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 10 percent erroneous returns?

4. Let Y have density function $f(y) = e^{-|y|}$. Compute the expectation, the variance and the moment generating function of the random variable Y. Find an expression for $\mu'_k = E[Y^k]$.

5. Find the smallest value of k in Tchebysheffs's theorem for which the probability that a random variable will take on a value between $\mu - k\sigma$ and $\mu + k\sigma$: (a) at least 68 %; (b) at least 95 %.

Math 447. 7–th Homework. First part of Chapter 5. Due Friday, November 19, 1999.

1. Let Y_1 and Y_2 be two jointly continuous random variables with joint density function have density function

$$f(y_1, y_2) = \begin{cases} cy_1 & \text{if } y_1, y_2 \leq 1 \leq y_1 + y_2, \\ 0 & \text{else.} \end{cases}$$

(a) Find the value of c.
(b) Find $P(Y_1 \geq 3/4, Y_2 \geq 1/2)$.
(c) Find $P(Y_1 \leq Y_2)$.

2. For the random variables in Exercise 1,
(a) Find the marginal densitites of Y_1 and Y_2.
(b) Find $P(Y_1 \leq 3/4 \mid Y_2 \leq 1/2)$.
(c) Find the conditional density of Y_1 given $Y_2 = y_2$, for $0 < y_2 < 1$.
(d) Find $P(Y_1 \leq 3/4 \mid Y_2 = 1/2)$.
(e) Are Y_1 and Y_2 independent random variables? Justify your answer.

3. Suppose that 3 batteries are randomly chosen from a group of 3 new, 4 used but still working, and 5 defective batteries. Let Y_1 and Y_2 denote, respectively, the number of new and used but still working batteries that are chosen.
(a) Find the joint probability function of Y_1 and Y_2.
(b) Represent this joint probability function as in the Table 5.2 in page 201 of the textbook.
(c) Find $P(Y_1 \geq Y_2)$.

4. For the random variables in Exercise 3,
(a) Find the marginal probability functions of Y_1 and Y_2.
(b) Find the probability function of $Y_1 + Y_2$.
(c) Find the conditional probability function of Y_1 given $Y_2 = 2$.
(d) Find and compare $P(Y_1 \leq 1)$ and $P(Y_1 \leq 1 \mid Y_2 = 1)$.
(e) Are Y_1 and Y_2 independent random variables? Justify your answer.

5. Let Y_1 and Y_2 be two jointly continuous random variables with joint density function have density function

$$f(y_1, y_2) = \begin{cases} y_1 e^{-(y_1+y_2)} & \text{if } y_1 > 0 \text{ ,} y_2 > 0, \\ 0 & \text{else.} \end{cases}$$

(a) Find the marginal probability density functions of Y_1 and Y_2.
(b) Are Y_1 and Y_2 independent random variables? Justify your answer.
(c) Find $P(Y_1 + Y_2 \leq 1)$.
(d) Find $P(Y_1 - Y_2 \geq 2)$.

Math 447. 8–th Homework. Second part of Chapter 5. Due Wednesday, December 1, 1999.

1. Let Y_1 and Y_2 be two jointly continuous random variables with joint density function

$$f(y_1, y_2) = \begin{cases} 24y_1y_2 & \text{if } 0 \le y_1, 0 \le y_2, y_1 + y_2 \le 1, \\ 0 & \text{else.} \end{cases}$$

(a) Find the mean and the variance of Y_1 and Y_2.
(b) Find the covariance and the correlation coefficient of Y_1 and Y_2.
(c) Find the mean and the variance of $U = -2 + 3Y_1 - 4Y_2$.
(d) Find $\text{Cov}(-2 + 3Y_1 - 4Y_2, -4 - 2Y_1 + 3Y_2)$.

2. For the random variables in Exercise 1, **(a)** Find the marginal densities of Y_1 and Y_2.
(b) Find the conditional density of Y_1 given $Y_2 = y_2$, for $0 < y_2 < 1$.
(c) Find $E[Y_1|Y_2 = y_2]$ and $V(Y_1|Y_2 = y_2)$, for $0 < y_2 < 1$.

3. The probabilities are 0.40, 0.50, and 0.10 that, in city driving, a certain kind of compact car will average less than 22 miles per gallon, from 22 to 26 miles per gallon, or more than 26 miles per gallon. Find the probability that among 10 such cars tested three will average less than 22 miles per gallon, six will average from 22 to 26 miles per gallon, and one will average more than 26 miles per gallon.

4. Let Y_1 and Y_2 have a bivariate normal distribuion with parameters $\mu_1 = 3$, $\mu_2 = 1$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$, and $\rho = \frac{3}{5}$. Determine the probabilities:
(a) $P(3 \le Y_2 \le 8)$.
(b) $P(3 \le Y_2 \le 8|Y_1 = 7)$.
(c) $P(-3 \le Y_1 \le 3)$.
(d) $P(-3 \le Y_1 \le 3|Y_2 = -4)$.

5. A community consists of 100 married couples. If during a given year 50 of the members of the community die, what is the expected number of marriages that remain intact? Assume that the set of people who die is equally likely to be any of the $\binom{200}{50}$ groups of size 50. (Hint: The number of marriages that remain intact Y satisfies $Y = Y_1 + \cdots + Y_{100}$, where

$$Y_i = \begin{cases} 1 & \text{if neither member of couple } i \text{ dies,} \\ 0 & \text{else.} \end{cases}$$

Math 447. First Midterm. Monday, October 4, 1999.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. From past experience a professor knows that the test score of a student taking her final examination is a random variable with mean 70 and variance 25. Assuming that the distribution of grades is approximately bell–shaped, what can be said about the probability that student's score is between 60 and 80?

2. In how many ways can 8 people be seated in a row if there are 5 men and they must sit next to each other?

3. What is the probability that each of the four players in a game of bridge receives one ace and one king?

4. A committee of 5 is chosen from a group of 8 men and 4 women. What is the probability the group contains a majority of women?

5. A track star runs two races on a certain day. The probability that he wins the first race is 0.7, the probability that he wins the second race is 0.6, and the probability that he wins both races is 0.5. Find the probability that he wins exactly one race.

6. Suppose that A, B and C are independent events and $P(A) = P(B) = P(C) = 1/3$. Find $P(A \cup B \cup C)$.

7. Stores A, B and C have 50, 75 and 100 employees and, respectively, 50, 60 and 70 percent of these are man. Resignations are equally likely among all employees, regardless of sex. One employee resigns, and this is a woman. What is the probability that she works in store A?

8. A ball is drawn from an urn containing 4 white and 2 black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes indefinitely. What is the probability that, of the first 8 balls drawn, exactly 3 are white?

9. If the probability function $p(y) = P(Y = y)$ of the random variable Y is given by $p(0) = 1/8$, $p(-4) = 1/4$, $p(2) = 1/2$, $p(-8) = 1/8$, $p(y) = 0$ for $y \notin \{0, 4, 2, -8\}$. Find the mean, the variance and the standard deviation of the random variable Y.

10. A gambling game is played as follows. A player, who pays \$4 to play the game tosses a fair coin 5 times. The player wins as many dollars as heads are tossed. Find the probability function of X, the player's net winnings. Find the expected value of the player's net winnings.

Math 447. Second Midterm. Wednesday, November 10, 1999.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. An experiment consists of tossing a fair die until a 6 occurs four times. What is the probability that the process ends after exactly ten tosses with a 6 occurring on the ninth and tenth tosses?

2. The number of imperfections in the weave of a certain textile has a Poisson distribution with a mean of four per square yard. The cost of repairing the imperfection in the weave is \$10 per imperfection. Find the mean and standard deviation of the repair cost for an 8–square–yard bolt of the textile.

3. Find the mean and the variance of the random variable X with cumulative distribution function given by

$$F(x) = \begin{cases} 0 & \text{if } x < -1, \\ 1/4 & \text{if } -1 \leq x < 2, \\ 3/4 & \text{if } 2 \leq x < 10, \\ 1 & \text{if } 10 \leq x. \end{cases}$$

4. The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} \frac{100}{x^2} & \text{if } x > 100, \\ 0 & \text{else,} \end{cases}$$

What is the probability that exactly 2 of 5 such tubes in radio set will have to be replaced within the first 150 hours of operation?

5. A Chamber of Commerce advertises that about 16% of the motels in town charge \$40 or more for a room and that the average price of a room is \$32. Assuming that room rates are approximately normal distributed, what is the variance in the room rates?

6. Find the density of a gamma random variable with mean 8 and variance 16.

7. Find the mean and the variance of a random variable Y with moment generating function

$$m(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}.$$

8. From past experience a professor knows that the test score of a student taking her final examination is a random variable with mean 70 and variance 25. What can be said about the probability that student's score is between 50 and 90 using the Tchebysheff's theorem?

9. Let Y_1 and Y_2 be two jointly continuous random variables with joint density function have density function

$$f(y_1, y_2) = \begin{cases} 3y_1 & \text{if } y_1 \leq 1, y_2 \leq 1, 1 \leq y_1 + y_2, \\ 0 & \text{else.} \end{cases}$$

Find $P(Y_1 \geq 3/4 \mid Y_2 \leq 1/2)$.

10. Find the marginal densitites of the random variables Y_1 and Y_2 in Exercise 9.

Math 447. Final. Tuesday, December 14, 1999.

Name: .... **Soc. Sec. No.** ....

Show all your work. No credit for lucky answers.

1. Suppose that a deck of 52 cards is shuffled and the top two cards are dealt. Find the probability that at least one ace is among the two cards.

2. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.

3. A student takes a multiple choice examination where each question has 5 possible answers. The student works a question correctly if he/she knows the answer, otherwise he/she guesses at random. Suppose that the student knows the answer to 70 % of questions. A question is chosen at random and it is found that the student got the right answer. What is the probability that the student actually knew the answer?

4. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = cx^9 e^{-x}, \quad x > 0.$$

Compute c and the expected lifetime of such a tube.

5. A supplier of heavy construction equipment has found that new customers are normally obtained through customer requests for a sales call and that the probability of a sale of particular piece of equipment is .3 If the supplier has three pieces of the equipment available for sale, what is the probability that it will take fewer than five customer contacts to clear the inventory?

6. The random variables Y_1 and Y_2 have joint density function

$$f(y_1, y_2) = \begin{cases} \frac{c}{\sqrt{y_1 y_2}}, & 0 < y_1 < 1,\ 0 < y_2 < 2, \\ 0 & \text{elsewhere.} \end{cases}$$

Find c and $P(Y_2 \geq 2Y_1)$

7. Let Y_1 and Y_2 be two random variables satisfying that

$$\mathrm{Var}(Y_1) = 1 \text{ and } \mathrm{Var}(Y_2) = \mathrm{Var}(2Y_1 - Y_2) = 4.$$

Find the covariance of Y_1 and Y_2.

8. The joint probability density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 6y_1, & 0 < y_1 < y_2 < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the marginal densities of Y_1 and Y_2. Are Y_1 and Y_2 independent random variables?

9. Let Y_1 denote the height in centimeters and Y_2 the weight in kilograms of male college students. Assume that Y_1 and Y_2 have a bivariate normal distribution with parameters $\mu_1 = 185$, $\sigma_1^2 = 100$, $\mu_2 = 84$, $\sigma_2^2 = 64$ and $\rho = 3/5$. Find $P(86.4 \leq Y_2 \leq 95.36 | Y_1 = 190)$.

10. Let Y be a random variable with density function

$$f(y) = \begin{cases} \frac{3y^2}{2}, & -1 < y < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the density of $U = \sqrt{5 - Y^3}$.

11. Let Y_1 and Y_2 have the joint density function

$$f(y_1, y_2) = \begin{cases} e^{-y_1 - y_2}, & 0 < y_1, y_2, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the density of $U = Y_1 + Y_2$.

12. The price asked Y_1 for a security is normally distributed with mean of \$50 and standard deviation of \$5. Buyers are willing to pay an amount Y_2 that it is normally distributed with a mean of \$45 and a standard deviation of \$2.5. Assuming that Y_1 and Y_2 are independent random variables, what is the probability that a transaction will take place?