Large samples tests

Let $Y_1, \ldots, Y_n$ be iid r.v's with mean θ, and known variance σ^2,

(1) To test $H_0: \theta = \theta_0$ (or $\theta \le \theta_0$) versus $H_a: \theta > \theta_0$

we reject H_0, if $\bar{y} \geq \theta_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$

the p-value is $P\left(N(0,1) \geq \frac{\sqrt{n}}{\sigma}(\bar{y}-\theta_0)\right)$

data, not random.

(2) To test $H_0: \theta = \theta_0$ (or $\theta \geq \theta_0$) versus $H_a: \theta < \theta_0$

we reject H_0 if $\bar{y} \le \theta_0 - z_\alpha \frac{\sigma}{\sqrt{n}}$

data

the p-value is $P\left(N(0,1) \le \frac{\sqrt{n}(\bar{y}-\theta_0)}{\sigma}\right)$

(3) To test $H_0: \theta = \theta_0$ versus $H_a: \theta \neq \theta_0$

we reject H_0, if $|\bar{y}-\theta_0| \geq z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

data

The p-value is $P\left(|N(0,1)| \geq \frac{\sqrt{n}|\bar{y}-\theta_0|}{\sigma}\right)$

Observe that $P_{H_0}(\text{reject } H_0) = \alpha$, in each case.

(1) $P_{\theta_0}\left(\bar{y} \geq \theta_0 + z_\alpha \frac{\sigma}{\sqrt{n}}\right) = \alpha$

and for each $\theta \le \theta_0$

$$P_\theta\left(\bar{y} \geq \theta_0 + z_\alpha \frac{\sigma}{\sqrt{n}}\right) = P\left(N(0,1) \geq \frac{\sqrt{n}(\theta_0-\theta)}{\sigma} + z_\alpha\right) \le \alpha$$

Similarly probabilities hold for (2) and (3)

10.6. The hourly wages in a particular industry are normally distributed with mean \$13.20 and standard deviation \$2.50. A company in this industry employs 40 workers, paying them an average of \$12.20 per hour. Can this company be accused of paying substandard wages? Use an $\alpha = 0.01$ level test.

$Y_1, \ldots Y_n \sim N(\theta, \sigma^2)$ $\quad \sigma^2 = (2.5)^2$

We test $H_0: \theta = 13.2$ versus $H_a: \theta < 13.2$

we reject H_0, if $\bar{Y} \leq \theta_0 - z_\alpha \frac{\sigma}{\sqrt{n}} = 13.2 - \frac{(2.325)(2.50)}{\sqrt{40}}$

$= 12.28$

Yes, the company pays substandard wages.

The p-value is

$P_{13.2}(\bar{Y} \leq 12.20) = P\left(N(0,1) \leq \frac{\sqrt{40}(12.20 - 13.2)}{2.5}\right)$

$= P(N(0,1) \leq -2.53) = 0.0057$

very strong evidency against H_0.

10.7. The output voltage for a certain electric circuit is specified to be 130. A sample of 40 independent readings on the voltage for this circuit gave a sample mean of 128.6 and a standard deviation of 2.1. Test the hypothesis that the average output voltage is 130 against the alternative that it is less than 130. Use a test with level 0.05.

$Y_1, \ldots Y_n$ iid $N(\theta, \sigma^2)$, σ^2 unknown

To test $H_0: \theta = 130$ versus $H_a: \theta < 130$

we reject H_0 if $\bar{Y} \le \theta_0 - \frac{z_\alpha S}{\sqrt{n}} = 130 - \frac{(1.645)\,2.1}{\sqrt{40}}$

$= 129.45$

We reject H_0

p-value is

$P_{130}(\bar{Y} \le 128.6) = P\left(N(0,1) \le \frac{\sqrt{40}(128.6-130)}{2.1}\right)$

$= P(N(0,1) \le -4.21) = 0.0000.$

10.8. The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of a certain type of steel, the Rockwell hardness index averaged 62 with standard deviation 8. The manufacturer claims that this type of steel has an average hardness index of at least 64. Is there sufficient evidence to refute the manufacturer's claim at the 1% significance level?

$Y_1 \ldots Y_n \sim N(\theta, \sigma^2)$, σ^2 unknown.

$H_0: \theta = 64$ versus $H_a: \theta < 64$

we reject H_0, if $\bar{Y} \le \theta_0 - z_\alpha \frac{s}{\sqrt{n}} = 64 - 2.325 \frac{8}{\sqrt{50}} = 61.3696$

No, there is no enough evidence to refute the manufacturer's claim

p-value

$P_{64}(\bar{Y} \le 62) = P\left(N(0,1) \le \frac{\sqrt{50}(62-64)}{8}\right)$

$= P(N(0,1) \le -1.7677) = 0.0384$

10.38 High airline occupancy rates on scheduled flights are essential to profitability. Suppose that a scheduled flight must average at least 60% occupancy to be profitable and that an examination of the occupancy rates for 120 10:00 am flights from Atlanta to Dallas showed mean occupancy rate per flight of 58% and standard deviation 11%. Test to see if sufficient evidence exists to support a claim that the claim is unprofitable. Find the p-value associated with the test. What would you conclude if you wished to implement the test at the level 0.10?

To test $H_0: \mu = 0.60$ versus $H_a: \mu < 0.60$

we reject H_0 if $\bar{Y} \le \mu_0 - z_\alpha \frac{s}{\sqrt{n}}$

The p-value is $P_{\mu_0}(\bar{Y} \le 0.58) = P\left(\frac{\bar{Y}-\mu_0}{s/\sqrt{n}} \le \frac{0.58-0.60}{0.11/\sqrt{120}} \right)$

$= P(N(0,1) \le -1.99) = 0.0233$

We reject H_0 at the level 0.10

The mean yield of corn in the United States is about 120 bushels per acre. A survey of 50 farmers this year gives a sample mean yield of $\bar{y} = 123.6$ bushels per acre. We want to know whether this is a good evidence that the national mean this year is not 120 bushels per acre. Assume that the farmers surveyed are a random sample from the population of all commercial corn growers and that the standandard deviation of the yield in this population is 10 bushels per acre.

Give the p-value for the test

$H_0: \mu = 120 \qquad H_a: \mu \neq 120$

$$\text{p-value} = P(|\bar{Y} - 120| \geq |123.6 - 120|)$$

$$= P\left(|N(0,1)| \geq \frac{\sqrt{50}}{10}|123.6 - 120|\right)$$

$$= 2P(N(0,1) \geq 2.5455) = 2(0.0055) = 0.011$$