Test for proportions

Let $Y \sim$ Binomial (n, p), where $0 \le p \le 1$ is an unknown parameter.

(1) To test $H_0: p = p_0$, versus $H_a: p > p_0$

we reject H_0 if $Y \geq np + z_\alpha \sqrt{np_0(1-p_0)}$

The p-value is $P\left(N(0,1) \geq \frac{Y - np_0}{\sqrt{np_0(1-p_0)}} \right)$ ← data

(2) To test $H_0: p = p_0$ versus $H_a: p < p_0$

we reject H_0, if $Y \le np_0 - z_\alpha \sqrt{np_0(1-p_0)}$

The p-value is $P\left(N(0,1) \le \frac{Y - np_0}{\sqrt{np_0(1-p_0)}} \right)$ ← data

(3) To test $H_0: p = p_0$ versus $H_a: p \neq p_0$

we reject H_0, if $|Y - np| \geq z_{\frac{\alpha}{2}} \sqrt{np_0(1-p_0)}$

The p-value is $P\left(|N(0,1)| \geq \frac{|Y - np_0|}{\sqrt{np_0(1-p_0)}} \right)$ ← data

10.13 According to a recent New York Times/CBS News Poll 60% of the 1429 adults interviewed were unable to name an elected official whom they admired. Is there sufficent evidence to claim that a majority of adults are unable to name an elected official whom they admire? Use a 0.01 level test.

$Y \sim \text{Binomial}(n, p)$ p = proportion of people which are unable to name..

We test $H_0: p = \frac{1}{2}$ versus $H_1: p > \frac{1}{2}$

we reject H_0 if $Y \geq np_0 + z_\alpha \sqrt{np_0(1-p_0)}$

$$Y \geq 1429(0.5) + 2.325\sqrt{1429 \tfrac{1}{2} \cdot \tfrac{1}{2}} = 758.44$$

$Y = (.60)(1429 = 857.4$

Yes, there is enough evidence to claim that a majority of adults are unable to name an elected official whom they admire

The p-value is

$$P(Y \geq 0.6) = P\left(N(0,1) \geq \frac{0.6 - 0.5}{\sqrt{\frac{0.5(0.5)}{1429}}}\right) = P(N(0,1) \geq 7.56) = 0.00$$

10.14 An article in the Washington Post stated that nearly 45% of all Americans have brown eyes. A random sample of 80 people found 32 with brown eyes. Is there sufficient evidence at the 0.01 level to indicate that the proportion of brown-eyed people in the region where the study was performed differs from the value reported in the Washington post?

Y = no. of brown-eyed people in the sample

We test $H_0: p = 0.45$ versus $H_a: p \neq 0.45$

We reject H_0 if $|Y - np_0| \geq z_{\frac{\alpha}{2}} \sqrt{np_0(1-p_0)}$

$= 2.575\sqrt{80(0.45)(0.55)} = 11.46$

$|Y - np_0| = |32 - 80(0.45)| = |-4| = 4$

Accept H_0.

The p-value is

$P(|Y - np_0| \geq |32 - 80(0.45)|)$

$= P\left(|N(0,1)| \geq \frac{|32 - 80(0.45)|}{\sqrt{80(0.45)(0.55)}}\right)$

$= 2P(N(0,1) \geq 0.90) = 0.3682$

10.43. A check-cashing service found that approximately 5% of all checks submitted to the service were bad. After instituting a check verification system to reduce its losses, the service found that only 45 checks were bad in a random sample of 1124 that were cashed. Does sufficient evidence exist to affirm that the check verification system reduced the proportion of bad checks? What attained significance level is associated with the test? What would you conclude at the $\alpha = 0.01$ level?

We test $H_0: p = 0.05$ versus $H_a: p < 0.05$

We reject H_0 if $\hat{p} \le p_0 - z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}}$

The p-value is $P_{p_0}\left(\hat{p} \le \frac{45}{1124}\right) = P\left(\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \le \frac{\frac{45}{1124} - 0.05}{\sqrt{\frac{(0.05)(1-0.05)}{1124}}}\right)$

$= P(N(0,1) \le -1.53) = 0.0630$

We accept H_0 at the $\alpha = 0.01$ level

In 1000 tosses of a coin, 560 heads and 440 tails appear. Is it reasonable to assume that the coin is fair? Do a two sided test at the level 0.05

$H_0: p = \frac{1}{2}$ $\quad H_a: p \neq \frac{1}{2}$

Reject H_0, if $|\hat{p} - p_0| \geq z_{\frac{\alpha}{2}} \sqrt{\frac{1}{n_0} p_0 (1-p_0)}$

$\hat{p} = \frac{560}{1000} = 0.56$

$|\hat{p} - p_0| = 0.06$

$z_{\frac{\alpha}{2}} \sqrt{\frac{1}{n} \hat{p}(1-\hat{p})} = 1.96 \sqrt{\frac{1}{1000} (0.50)(0.50} = 0.03010$

Reject H_0

The p-value is

$P(|Y - np_0| \geq |560 - (1000)(0.5)|$

$= P(|N(0,1)| \geq \frac{|560 - (1000)(0.5)|}{\sqrt{1000 (0.5)(0.5)}}$

$= 2P(N(0,1) \geq 3.79) = 0.000$

10.42 Do you believe that an exceptionally high percentage of the executives of large corporations are right-handed? Although 85% of the general public is right-handed, a survey of 300 chief executive officers of large corporations found that 96% were right-handed.

a. Is this difference in percentages statistically significant? Test using $\alpha = 0.06$

b. Find the p-value for the test, and explain what it means.

$H_0: p = 0.85$ $\quad H_a: p > 0.85$.

We reject H_0 if $\hat{p} \geq p_0 + z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}} = 0.85 + 2.326 \sqrt{\frac{(0.85)(1-0.85)}{300}}$

$= 0.8979.$

Yes, the difference in percentages is statistically significant

b. The p-value is

$$P_{p_0}(\hat{p} \geq 0.96) = P\left(\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \geq \frac{0.96 - 0.85}{\sqrt{\frac{(0.85)(0.15)}{300}}} \right)$$

$$= P(N(0,1) \geq 5.33) = 0.00\ldots$$