10.31 In Exercise 10.24, how large should the sample size be to permit $\alpha = 0.01$ and $\beta = 0.05$ when $\mu_a = 5.5$?

$\mu_0 = 5$. $s = 3.1$

We test $H_0: \mu_0 = 5$ versus $H_a: \mu_a = 5.5$

We reject H_0, if

$$\bar{Y} \geq \mu_0 + z_\alpha \frac{s}{\sqrt{n}} = 5 + \frac{2.326\,(3.1)}{\sqrt{n}}$$

$$\beta = 0.05 = P_{\mu_a}\left(\bar{Y} \leq \mu_0 + z_\alpha \frac{s}{\sqrt{n}}\right) = P_{\mu_a}\left(\bar{Y} \leq 5 + \frac{(2.326)(3.1)}{\sqrt{n}}\right)$$

$$= P\left(N(0,1) \leq \frac{5 + \frac{(2.326)(3.1)}{\sqrt{n}} - 5.5}{3.1/\sqrt{n}}\right)$$

$$= P\left(N(0,1) \leq 2.326 - \frac{(0.5)\sqrt{n}}{3.1}\right) = P\left(N(0,1) \geq -2.326 + \frac{(0.5)\sqrt{n}}{3.1}\right)$$

$$= P(N(0,1) \geq 1.645)$$

$$1.645 = -2.326 + \frac{(0.5)\sqrt{n}}{3.1}$$

$$n = \left(\frac{3.1\,(1.645 + 2.326)}{0.5}\right)^2 = 605.84$$

10.32 Refer to Exercise 10.31. Find the sample sizes that give $\alpha = 0.05$ $\beta = 0.05$ when $\mu_1 - \mu_2 = 3$
(Assume equal-size samples for this group).

Reject H_0 if $\bar{Y}_1 - \bar{Y}_2 \geq z_\alpha \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = 1.645 \sqrt{\frac{(4.34)^2 + (4.56)^2}{n}}$

$$= \frac{(1.645)(6.2952)}{\sqrt{n}}$$

$P(N(0,1) \geq 1.645)$

$$0.05 = P\left(\bar{Y}_1 - \bar{Y}_2 \leq \frac{(1.645)(6.2952)}{\sqrt{n}} \,\middle|\, \mu_1 - \mu_2 = 3\right)$$

$$= P\left(N(0,1) \leq \frac{\frac{(1.645)(6.2952)}{\sqrt{n}} - 3}{\frac{6.2952}{\sqrt{n}}}\right)$$

$$= P\left(N(0,1) \leq 1.645 - \frac{3\sqrt{n}}{6.2952}\right)$$

$$= P\left(N(0,1) \geq -1.645 + \frac{3\sqrt{n}}{6.2952}\right)$$

$$1.645 = -1.645 + \frac{3\sqrt{n}}{6.2952}$$

$$n = \left(\frac{(1.645 + 1.645)(6.2952)}{3}\right)^2 = 47.66$$

10.33 A random sample of 37 second-graders who participated in sports had dexterity scores with mean 32.19 and standard deviation 4.34. An independent sample of 37 second-grader who did not participate in sports had manual dexterity scores with mean 31.68 and standard deviation 4.56.

(a) Test to see whether sufficient evidence exists to indicate that second-graders who participate in sports have a higher mean dexterity score. Use $\alpha = 0.05$

(b) For the rejection region used in (a), calculate β when $\mu_1 - \mu_2 = 3$

$H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 > \mu_2$

Reject H_0, if $\dfrac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \geq z_\alpha = 1.645$

$$= \frac{32.19 - 31.68}{\sqrt{\frac{(4.34)^2}{37} + \frac{(4.56)^2}{37}}} = 0.4927$$

H_0 is not rejected

(b) $\beta = P_{\mu_1 - \mu_2}(\bar{Y}_1 - \bar{Y}_2 \leq 1.645) = P\left(N(0,1) \leq \dfrac{1.645\sqrt{\frac{(4.34)^2}{37} + \frac{(4.56)^2}{37}} - 3}{\sqrt{\frac{(4.34)^2}{37} + \frac{(4.56)^2}{37}}}\right)$

$= P(N(0,1) \leq -1.253) = 0.1056$

10.34 Refer to Exercise 10.33, Find the sample sizes that gives $\alpha = 0.05$ and $\beta = 0.05$ when $\mu_1 - \mu_2 = 3$ Assume equal-size samples for each group

$$\alpha = P_{\mu_1-\mu_2=0}(\bar{Y}_1 - \bar{Y}_2 > K) = P\left(N(0,1) \geq \frac{K}{\sqrt{\frac{(4.34)^2+(4.56)^2}{n}}}\right) = 0.05$$

$$1.645 = \frac{\sqrt{n}\,K}{6.2951}$$

$$\beta = P_{\mu_1-\mu_2}(\bar{Y}_1 - \bar{Y}_2 < K) = P\left(N(0,1) \leq \frac{K-3}{\sqrt{\frac{(4.34)^2+(4.56)^2}{n}}}\right) = 0.05$$

$$-1.645 = \frac{\sqrt{n}(K-3)}{6.2951}$$

$$2(1.645) = \frac{3\sqrt{n}}{6.2951}$$

$$n = \left(\frac{(2)(1.645)(6.2951)}{3}\right)^2 = 47.66$$