Small sample hypothesis testing for μ and $\mu_1-\mu_2$

If $Y_1, \dots, Y_n$ constitute a random sample from a normal distribution

To test $H_0: \mu = \mu_0$ versus $H_a: \mu > \mu_0$

we reject H_0 if $\bar{Y} \geq \mu_0 + t_\alpha \frac{S}{\sqrt{n}}$

To test $H_0: \mu = \mu_0$ versus $H_a: \mu < \mu_0$

we reject H_0 if $\bar{Y} \leq \mu_0 - t_\alpha \frac{S}{\sqrt{n}}$

To test $H_0: \mu = \mu_0$ versus $H_a: \mu \neq \mu_0$

we reject H_0 if $|\bar{Y} - \mu_0| > t_\alpha \frac{S}{\sqrt{n}}$

Small-sample tests for comparing two population means

Assume independent samples from normal distributions with $\sigma_1^2 = \sigma_2^2$

To test $H_0: \mu_1 - \mu_2 = D_0$ versus $H_1: \mu_1 - \mu_2 > D_0$

we reject H_0 if $\bar{Y}_1 - \bar{Y}_2 \geq D_0 + t_\alpha S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

where $S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$

To test $H_0: \mu_1 - \mu_2 = D_0$ versus $H_a: \mu_1 - \mu_2 < D_0$

we reject H_0 if $\bar{Y}_1 - \bar{Y}_2 \leq D_0 - t_\alpha S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

To test $H_0: \mu_1 - \mu_2 = D_0$ versus $H_a: \mu_1 - \mu_2 \neq D_0$

we reject H_0 if $|\bar{Y}_1 - \bar{Y}_2 - D_0| \geq t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

10.51 A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805 790, 793 and 802 tons. Do these data indicate that the average yield is less than 800 tons, and hence that something is wrong with the process? Test at the 5% level of significance. What assumptions must be satisfied in order for the procedure you used to analyze these data to be valid? Give bounds for the associated p-value.

785, 805, 790, 793, 802,

$\sum Y_i = 3975 \qquad \bar{y} = \frac{\sum Y_i}{n} = 795$

$\sum Y_i^2 = 3160403$

$\sum (Y_i - \bar{Y})^2 = \sum Y_i^2 - n(\bar{Y})^2 = 3160403 - 5(795)^2 = 278$

$s^2 = \frac{1}{n-1} \sum (Y_i - \bar{Y})^2 = 69.5$

$H_0: \mu = 800 \qquad H_a: \mu < 800$

We reject H_0, if $\bar{Y} \leq \mu_0 - t_\alpha \frac{s}{\sqrt{n}} = 800 - \frac{2.132\sqrt{69.5}}{\sqrt{5}} = 792.05$.

We accept to H_0

The data does not indicate that the average yield is less than 800 tons

The p-value is

$P(\bar{Y} \leq 795) = P\left(t(4) \leq \frac{795-800}{\sqrt{69.5}/\sqrt{5}}\right) = P(t(4) \leq -1.34)$

$= P(t(4) \geq 1.34) \qquad P(t(4) \geq 1.533) = 0.1$

The p-value is ≥ 0.1

10.52 A coin-operated soft drink machine was designed to discharge on the average, 7 ounces of beverage per cup. In a test of the machine ten cupfuls of beverage were drawn from the machine and measured. The mean and standard deviation of the ten measurements were 7.1 ounces and 0.12 ounces, respectively. Do these data present sufficent evidence to indicate that the mean discharge differs from 7 ounces? What can be said about the attained significance level for this test? What is the appropiate decision if $a = 0.10$?

$H_0: \mu = 7 \quad H_a: \mu \neq 7.$

$\bar{y} = 7.1 \quad s = 0.12$

The p-value of the test is

$P(|\bar{y} - \mu_0| \geq 0.1) = P\left(\frac{\sqrt{n}|\bar{y}-\mu_0|}{s} \geq \frac{\sqrt{10}\,(0.1)}{0.12} \right) = 2P(t(9) \geq 2.635)$

$P(t(9) \geq 2.262) = 0.025$

$P(t(9) \geq 2.821) = 0.010$

$0.02 \leq$ p-value ≤ 0.05

Reject H_0

10.54. Researchers have shown that cigarette smoking has a deleterious effect on lung function. In their study of the effect of cigarette smoking on the carbon monoxide diffusing capacity (DL) of the lung Ronald Knudson, Walter Kaltenborn and Benjamin Burrows found that current smokers had DL readings significantly lower than either ex-smokers or non-smokers. The carbon monoxide diffusing capacity for a random sample of current smokers was as follows:

103.768	88.602	73.003	123.086	91.052
92.295	61.675	90.677	84.023	76.014
100.615	88.017	71.210	82.115	89.222
102.754	108.579	73.154	104.755	90.479

Do these data indicate that the mean DL reading for current smokers is lower than 100, the average DL reading for nonsmokers? Test at the $\alpha = 0.01$ level. What is the p-value associated with this test?

$H_0: \mu = 100$ versus $H_a: \mu < 100$

$\sum y_i = 1797.095$ $\quad \sum y_i^2 = 165,697.7081$

$$\bar{y} = \frac{1797.095}{20} = 89.85475$$

$$\sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 4220.18627$$

$$s^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2 = 222.115067$$

Reject H_0 if $\frac{\bar{y}-\mu}{s/\sqrt{n}} \le -t_{0.01} = -2.539$

$= -3.05$

We reject H_0. $p\text{-value} < 0.005$

10.57. Two methods for teaching reading were applied to randomly selected groups of elementary schoolchildren and then compared on the basis of a reading comprehension test given at the end of the learning period.

	Method I	Method II
Number of children in group	11	14
$\bar{Y}$	64	69
s^2	52	71

Do the data present sufficient evidence to indicate a difference in the mean scores for the populations associated with the two teaching methods?
What can be said about the attained significance level?
What assumptions are required?
What would you conclude at the $\alpha = 0.05$ level of significance?

$$\frac{|\bar{Y}_1 - \bar{Y}_2|}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{|64-69|}{\sqrt{62.7391}\sqrt{\frac{1}{11}+\frac{1}{14}}} = 1.5667$$

$$S_p^2 = \frac{(10)(52)+(13)(71)}{23} = 62.7391$$

The p-value is $P(|t(23)| \geq 1.5667) = 2P(t(23) \geq 1.5667)$

$P(t(23) \geq 1.319) = 0.10 \qquad P(t(23) \geq 1.714) = 0.05$

$0.10 < \text{p-value} < 0.2$

Do not reject H_0 for $\alpha = 0.05$

We must assume that the data come from two normal populations with a common variance.

10.60 The strength of concrete depends, to some extend, on the method used for drying it. Two different drying methods yielded the results shown in the accompanying table for independently tested specimens (measurements in psi)

Method I	Method II
$n_1 = 7$	$n_2 = 10$
$\bar{y}_1 = 3250$	$\bar{y}_2 = 3240$
$s_1 = 210$	$s_2 = 190$

Do the methods appear to produce concrete with different mean strengths? Use $\alpha = 0.05$. What is the attained significance level?

$H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 \neq \mu_2$

$$t = \frac{\bar{y}_1 - \bar{y}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{3250-3240}{\sqrt{39300}\sqrt{\frac{1}{7}+\frac{1}{10}}} = 0.1023$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{6(210)^2 + 9(190)^2}{15} = 3930$$

p-value $= P(|t(15)| \geq 0.1023) = 2P(t(15) \geq 0.1023)$

$P(t(15) \geq 1.341) = 0.10 \qquad$ p-value ≥ 0.20

Do not reject H_0