Testing hypothesis concerning variances

Let $Y_1, \dots, Y_n$ denote a random sample from a normal distribution with unknown mean μ and unknown variance σ^2.

S^2 is an unbiased estimator of σ^2

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

(1) To test $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 > \sigma_0^2$

we reject H_0, if $S^2 \geq \frac{\sigma_0^2}{(n-1)} \chi^2_\alpha$

the p-value is $P(S^2 \geq \overset{\swarrow \text{data}}{s^2})$

(2) To test $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 < \sigma_0^2$

we reject H_0, if $S^2 \leq \frac{\sigma_0^2}{(n-1)} \chi^2_{1-\alpha}$

the p-value is $P(S^2 \leq \overset{\swarrow \text{data}}{s^2})$

(3) To test $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 \neq \sigma_0^2$

we reject H_0, if $S^2 \leq \frac{\sigma_0^2}{(n-1)} \chi^2_{1-\frac{\alpha}{2}}$ or $S^2 \geq \frac{\sigma_0^2}{(n-1)} \chi^2_{\frac{\alpha}{2}}$

10.66 A manufacturer of hard safety hats for construction workers is concerned about the mean and the variation of the forces its helmets transmit to wearers when subjected to a standard external force. The manufacturer desires the mean force transmitted by helmets to be 800 pounds (or less) well under the legal 1000 pound-limit, and desires σ to be less than 40. Tests were run on a random sample of $n=40$ helmets, and the sample mean and variance were found to be equal to 825 pounds and 2350 pounds^2 respectively.

a. If $\mu = 800$ and $\sigma = 40$, is it likely that any helmet subjected to the standard external force will transmit a force to a wearer in excess of 1000 pounds? Explain.

b. Do the data provide sufficient evidence to indicate that when subjected to the standard external force, the helmets transmit a mean force exceeding 800 pounds?

c. Do the data provide sufficient evidence to indicate that σ exceeds 40?

a. Y = force transmitted to the weaver.

$$P(Y \geq 1000) = P\left(\frac{Y-800}{40} \geq \frac{1000-800}{40}\right) = P(N(0,1) \geq 5) = 0.00..$$

b. We test $H_0: \mu = 800$ versus $H_a: \mu > 800$, the p-value is

$$P(\bar{Y} \geq 825) = P\left(\frac{\bar{Y}-\mu_0}{S/\sqrt{n}} \geq \frac{825-800}{\sqrt{2350}/\sqrt{40}}\right) = P(N(0,1) \geq 3.26) = 0.000.$$

Reject H_0

c. We test $H_0: \sigma^2 = 40^2$ versus $H_a: \sigma^2 > 40^2$, the p-value is

$$P(S^2 \geq 2350) = P\left(\frac{(n-1)S^2}{\sigma_0^2} \geq \frac{(39)(2350)}{40^2}\right) = P(\chi^2(39) \geq 57.281)$$

$$P(\chi^2(40) \geq 55.75) = 0.05 \qquad P(\chi^2(40) \geq 59.34) = 0.025$$

$$0.05 > \text{p-value} > 0.025$$

Reject H_0

10.67 A manufacturer of a machine to package soap powder claimed that her machine could load cartoons at a given weight with a range of no more than 0.4 ounce. The mean and variance of a sample of eight 3-pound boxes were found to equal 3.1 and 0.018, respectively. Test the hypothesis that the variance of the population of weight measurements is $\sigma^2 = 0.01$ against the alternative that it is $\sigma^2 > 0.01$. Use an $\alpha = 0.05$ level of significance.

What assumptions are required for this test?

What can be said about the attained significance level?

The p-value is $P(S^2 \geq 0.018) = P\left(\frac{(n-1)S^2}{\sigma_0^2} \geq \frac{(7)(0.018)}{0.01}\right)$

$P(X^2(7) \geq 12.6)$

$P(X^2(7) \geq 12.0170) = 0.10$ $\quad P(X^2(7) \geq 14.06) = 0.05$

$0.10 > p\text{-value} > 0.05$

Accept Ho at the level 0.05.

The assumptions required for this test is that the distribution is normal

10.73. A precision instrument is guaranteed to be accurate to within 2 units. A sample of four instrument readings on the same object yielded the measurements 353, 351, 351 and 355.
Give the attained significance level for testing the null hypothesis that $\sigma = 0.7$ versus the alternative that $\sigma > 0.7$

We test $H_0: \sigma^2 = 0.49$ versus $H_a: \sigma^2 > 0.49$

$\sum Y_i = 1410 \qquad \sum Y_i^2 = 497036$

$$s^2 = \frac{\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}}{n-1} = \frac{497036 - \frac{(1410)^2}{4}}{3} = 3.667$$

Reject H_0 if $\frac{(n-1)s^2}{\sigma_0^2} \geq \chi^2_\alpha$

$$= \frac{(3)(3.667)^2}{0.49} = 22.45$$

$\chi^2_{0.005} = 12.8381 \qquad$ p-value < 0.005