Tests involving the variances of two samples

$Y_{11}, \dots, Y_{1n_1}$ are iid r.v's from $N(\mu_1, \sigma_1^2)$

$Y_{21}, \dots, Y_{2n_2}$ are iid r.v's from $N(\mu_2, \sigma_2^2)$

The two samples are independent

(1) To test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 > \sigma_2^2$

we reject H_0 if $\frac{s_1^2}{s_2^2} \geq F_{n_2-1}^{n_1-1}(\alpha)$

data

The p-value is $P\left(\frac{S_1^2}{S_2^2} \geq \frac{s_1^2}{s_2^2} \right)$

(2) To test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 < \sigma_2^2$

we reject H_0 if $\frac{s_1^2}{s_2^2} \leq F_{n_2-1}^{n_1-1}(1-\alpha)$

(3) To test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 \neq \sigma_2^2$

we reject H_0 if either $\frac{s_1^2}{s_2^2} \geq F_{n_2-1}^{n_1-1}\left(\frac{\alpha}{2}\right)$ or $\frac{s_1^2}{s_2^2} \leq F_{n_2-1}^{n_1-1}\left(1-\frac{\alpha}{2}\right)$

Since $F_m^n \overset{d}{=} \frac{1}{F_n^m}$, $F_m^n(\alpha) = \frac{1}{F_n^m(1-\alpha)}$

10.72 The closing prices of two common stocks were recorded for a period of 16 days. The means and variances were

$$\bar{Y}_1 = 40.33 \quad \bar{Y}_2 = 42.54 \quad S_1^2 = 1.54 \quad S_2^2 = 2.96$$

Do these data present sufficient evidence to indicate a difference in variability of closing prices of the two stocks for the populations associated with the two samples? Give bounds for the attained significance level. What would you conclude, with $\alpha = 0.02$?

To test $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 \neq \sigma_2^2$

Reject H_0 if $\frac{S_1^2}{S_2^2} = \frac{1.54}{2.96} = 0.520 \geq F_{n_2-1}^{n_1-1}\left(\frac{\alpha}{2}\right)$

or $\frac{S_1^2}{S_2^2} \leq F_{n_2-1}^{n_1-1}\left(1-\frac{\alpha}{2}\right) = \frac{1}{F_{n_1-1}^{n_2-1}\left(\frac{\alpha}{2}\right)}$

$$F_{n_1-1}^{n_2-1}\left(\frac{\alpha}{2}\right) \leq \frac{S_2^2}{S_1^2} = 1.9220$$

For $\alpha = 0.20$, we reject H_0 if

either $0.520 \geq 1.97$

or $1.97 \leq 1.92$

α	F_{15}^{15}
0.10	1.97
0.05	2.40
0.025	2.86
0.010	3.52
0.005	4.07

We reject H_0 for $\alpha = 0.20$

The p-value is > 0.20

Example

In comparing the variability of the tensile strength of two kinds of structural steel, an experiment yielded the following results:

$n_1 = 13$, $s_1^2 = 19.2$ $n_2 = 16$ and $s_2^2 = 3.5$

where the units of measurements are 1000 pounds per square inch. Assuming that the measurements constitute independent random samples from two normal populations, test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against the alternative $\sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance

Reject H_0 if $\frac{S_1^2}{S_2^2} \geq F_{n_2-1}^{n_1-1}\left(\frac{\alpha}{2}\right)$ or $\frac{S_1^2}{S_2^2} \leq F_{n_2-1}^{n_1-1}\left(1-\frac{\alpha}{2}\right)$

$$\frac{S_1^2}{S_2^2} = \frac{19.2}{3.5} = 5.4857$$

$$F_{15}^{12}(0.01) = 3.67 \qquad F_{n_2-1}^{n_1-1}\left(1-\frac{\alpha}{2}\right) = \frac{1}{F_{n_1}^{n_2-1}\left(\frac{\alpha}{2}\right)} = \frac{1}{F_{12}^{15}(0.01)} = \frac{1}{4.01} = 0.2494$$

Reject H_0 if $5.48 \geq 3.67$ or $5.48 \leq 0.2494$

We reject H_0 at the level 0.02

10.75. Refer to Exercise 10.58.
(amounts of chemical residue found in the brain tissue of brown pelicans)

Juveniles	Nestlings
$n_1 = 10$	$n_2 = 13$
$\bar{y}_1 = 0.041$	$\bar{y}_2 = 0.026$
$s_1 = 0.017$	$s_2 = 0.006$

Is there sufficient evidence, at the 5% significance level to support concluding that the variance in measurements of DDT levels is greater for juveniles that it is for nestlings?

$H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 > \sigma_2^2$

The test statistic is $F = \dfrac{S_1^2}{S_2^2} = \dfrac{(0.017)^2}{(0.006)^2} = 8.03$

The rejection region with $\alpha = 0.05$ is $F > F^{9}_{12,0.05} > 2.80$

and H_0 is rejected

We conclude that $\sigma_1^2 > \sigma_2^2$

α	F^{9}_{12}
0.10	2.21
0.05	2.80
0.025	3.44
0.010	4.39
0.005	5.20

p-value < 0.005