7-43 (782x1001x24b jpeg)

7.56. The quality of computer disks is measured by the number of missing pulses. Brand δ is such that 80% of the disks have no missing pulses. If 100 disks of brand δ are inspected, what is the probability that 15 or more contain missing pulses

S = no. of disks which have no missing pulses

$S \sim Bin(100, 0.80)$ $\quad E[S] = 80 \quad Var(S) = 16$

$$P(S \le 85) \simeq P\left(N(0,1) \le \frac{85-80}{\sqrt{16}}\right) = P(N(0,1) \le 1.25)$$

$$= 1 - 0.1056 = .8944$$

$$P(S \le 85) = \sum_{j=0}^{85} \binom{100}{j} (0.8)^j (0.2)^{85-j} = 0.9196$$

$$P(S \le 85.5) = P\left(N(0,1) \le \frac{85.5-80}{\sqrt{16}}\right) = P(N(0,1) \le 1.375)$$

$$= 0.9154$$

$$P(S < 86) = P\left(N(0,1) \le \frac{86-80}{4}\right) = P(N(0,1) \le 1.5)$$

$$= 0.9332$$

$$P(S \le 85.6098) = P\left(N(0,1) \le \frac{85.6098-80}{4}\right)$$

$$= P(N(0,1) \le 1.4024) = 0.9196$$

7-44 (807x559x24b jpeg)

7.34 A machine is shut down for repairs if a random sample of 100 items selected from the daily output of the machine reveals at least 15% defectives. If on given day the machine is producing only 10% defective items, what is the probability that it will be shut down?

Y = no. of defectives in the random sample of 100 items.

$P(\text{the machine is shut down}) = P(Y \geq 15)$

$$= P(Y \geq 14.5) = P\left(\frac{Y - 100(0.10)}{\sqrt{100(0.10)(0.9)}} \geq \frac{14.5 - 100(0.10)}{\sqrt{100(0.10)(0.9)}} \right)$$

$$\simeq P(Z \geq 1.5) = 0.0668$$

7.19. An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.5 inches if she randomly samples 100 men, find the probability that the difference between the sample mean and the true population will not exceed 0.5 inch.

$$P(|\bar{Y}-\mu| \le 0.5) = P(-0.5 \le \bar{Y}-\mu \le 0.5) =$$

$$= P\left(-\frac{0.5\sqrt{100}}{2.5} \le \frac{\sqrt{n}(\bar{Y}-\mu)}{\sigma} \le \frac{0.5\sqrt{100}}{2.5}\right) = P(-2 \le N(0,1) \le 2)$$

$$= 1-2P(N(0,1) \ge 2) = 1-2(0.0228) = 0.9544$$

7.20 Suppose that the antropologist of Exercise 7.19 wants the difference between the sample mean and the population mean to be less than 0.4 inches with probability 0.95. How many men should she sample to achieve this objective?

$$0.95 = P(|\bar{Y}-\mu| \le 0.4) = P(-0.4 \le \bar{Y}-\mu \le 0.4)$$

$$= P\left(-\frac{0.4\sqrt{n}}{2.5} \le \frac{\sqrt{n}(\bar{Y}-\mu)}{\sigma} \le \frac{0.4\sqrt{n}}{2.5}\right) = P\left(-\frac{0.4\sqrt{n}}{2.5} \le N(0,1) \le \frac{0.4\sqrt{n}}{2.5}\right)$$

$$= 1-2P\left(N(0,1) \ge \frac{0.4\sqrt{n}}{2.5}\right) \qquad P\left(N(0,1) \ge \frac{0.4\sqrt{n}}{2.5}\right) = \frac{1-0.95}{2} = 0.025$$

$$\frac{0.4\sqrt{n}}{2.5} = 1.96 \qquad n = \left(\frac{(2.5)(1.96)}{0.4}\right)^2 = 150.0625$$

7-46 (850x1085x24b jpeg)

7.54. Suppose that $Y_1, Y_2, \ldots Y_{40}$ denote a random sample of measurements on the proportion of impurities in iron ore samples. Let each variable Y_i have the probability density function given by

$$f(y) = \begin{cases} 3y^2 & 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

The ore is to be rejected by the potential buyer if $\bar{Y}$ exceeds 0.7. Find $P(\bar{Y} \geq 0.7)$ for the sample of size 40.

$Y \sim \text{Beta}(\alpha, \beta)$

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1} \qquad E[Y] = \frac{\alpha}{\alpha+\beta} \qquad V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$\alpha = 3 \quad \beta = 1$

$$E[Y] = \frac{3}{3+1} = \frac{3}{4} \qquad V(Y) = \frac{3 \cdot 1}{4^2 \cdot 5} = \frac{3}{80}$$

$$E[\bar{Y}] = \frac{3}{4} = 0.75 \qquad V(\bar{Y}) = \frac{3}{80 \cdot 40} = \frac{3}{3200}$$

$$P(\bar{Y} \geq 0.7) = P\left(Z \geq \frac{0.7 - 0.75}{\sqrt{\frac{3}{3200}}}\right) = P(Z \geq -1.631)$$

$$= 1 - P(Z \geq 1.63) = 1 - 0.0516 = 0.9484$$