7-21 (778x955x24b jpeg)

Example 7.1 page 332

A bottling machine can be regulated so that it discharges an average of μ ounces per bottle. It has been observed that the amount of fill dispensed by the machine is normally distributed with $\sigma = 1.00$ ounce. A sample of $n = 9$ filled bottles is random selected from the output of the machine on a given day and the ounces of fill measured for each. Find the probability that the sample mean will be within 0.3 ounce of the true mean μ for that particular setting

Let $Y_1 \ldots Y_9$ be the ounces of fill

$Y_1 \ldots Y_9$ are iid r.vs $N(\mu, \sigma^2)$ $\sigma = 1$

$\bar{Y} \sim N(\mu, \frac{\sigma^2}{n} = \frac{1}{9})$

$P(|\bar{Y} - \mu| \le 0.3) = P(-0.3 \le \bar{Y} - \mu \le 0.3)$

$$= P\left(-\frac{0.3}{\sqrt{\frac{1}{9}}} \le \frac{\bar{Y} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \le \frac{0.3}{\sqrt{\frac{1}{9}}}\right) = P(-0.9 \le N(0,1) \le 0.9)$$

$$= 1 - P(N(0,1) > 0.9) = 1 - 2\,(0.1841) = 0.6318$$

7-22 (761x580x24b jpeg)

Example 7.2

How many observations should be included in the sample if we wish $\bar{Y}$ to be within 0.3 ounce of μ with probability 0.95

$$0.95 = P(|\bar{Y}-\mu| \leq 0.3) = P(-0.3 < \bar{Y}-\mu < 0.3)$$

$$= P\left(\frac{-0.3}{\frac{1}{\sqrt{n}}} \leq \frac{\bar{Y}-\mu}{\frac{\sigma^2}{\sqrt{n}}} \leq \frac{0.3}{\frac{1}{\sqrt{n}}}\right) = 1-2P(N(0,1) \geq 0.3\sqrt{n})$$

$$0.025 = P(N(0,1) \geq 0.3\sqrt{n})$$

$$1.96 = 0.3\sqrt{n} \qquad n = \left(\frac{1.96}{0.3}\right)^2 = 42.68$$

7-23 (807x934x24b jpeg)

7.11 Refer to Exercise 7.5. Suppose that $n=20$ observations are to be taken on $\ln(LC50)$ measurements and that $\sigma^2 = 1.4$. Let s^2 denote the sample variance of 20 measurements.

(a) Find a number b such that $P(S^2 \le b) = 0.975$

(b) Find a number a such that $P(a \le S^2) = 0.975$.

(c) If a and b are as in the previous parts, what is $P(a \le S^2 \le b$

(a) $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)$

$0.975 = P(s^2 \le b) = P\left(\chi^2(19) \le \frac{19b}{1.4}\right)$

$0.025 = P\left(\chi^2(19) \ge \frac{19b}{1.4}\right)$ $\quad \frac{19b}{1.4} = 32.8523 \quad b = 2.42$

(b) $0.975 = P(s^2 \le a) = P\left(\chi^2(19) \le \frac{19a}{1.4}\right)$

$\frac{19a}{1.4} = 8.90655 \qquad a = 0.656$

(c) $P(a \le s^2 \le b) = 0.95$

7.10

(a) If $U \sim \chi^2(\nu)$, find $E[U]$ and $V(U)$

The density of U is $f(y) = \dfrac{y^{\frac{\nu}{2}-1} e^{-y/2}}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}$

$$E[U] = \int_0^\infty \frac{u^{\frac{\nu}{2}} e^{-u/2}}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}\,du = \frac{2^{\frac{\nu+2}{2}}\,\Gamma(\frac{\nu+2}{2})}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} = 2\,\frac{\nu}{2} = \nu$$

$$E[U^2] = \int_0^\infty \frac{u^{\frac{\nu+2}{2}} e^{-u/2}}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}\,du = \frac{2^{\frac{\nu+4}{2}}\,\Gamma(\frac{\nu+4}{2})}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} = 4\left(\frac{\nu}{2}+1\right)\frac{\nu}{2}$$

$= \nu(\nu+2)$

$V(U) = 2\nu$

(b) Find $E[S^2]$ and $V(S^2)$

$\frac{(n-1)S^2}{\sigma^2} = Y \sim \chi^2(n-1)$

$E\left[\frac{(n-1)S^2}{\sigma^2}\right] = n-1 \quad E[S^2] = \sigma^2$

$V\left(\frac{(n-1)S^2}{\sigma^2}\right) = \frac{(n-1)^2}{\sigma^4} V(S^2) = 2(n-1)$

$V(S^2) = \frac{2\sigma^4}{n-1}$