Some common unbiased point estimators

A. Let $Y_1 \ldots Y_n$ be iid $N(\mu, \sigma^2)$

1. $\bar{Y}$ estimates μ

   $E[\bar{Y}] = \mu \qquad V(\bar{Y}) = \frac{\sigma^2}{n}$

2. $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \bar{Y})$ = sample variance estimates σ^2

   $E[S^2] = \sigma^2 \qquad V(S^2) = \frac{2\sigma^4}{n-1}$

   $\frac{(n-1)}{\sigma^2} S^2 \sim \chi^2(n-1)$

B. Let Y be the number of successes in n independent trials with probability of success p. Y has a binomial distribution

$\hat{p} = \frac{Y}{n}$ estimates p

$E[\hat{p}] = p \qquad V(\hat{p}) = \frac{p(1-p)}{n}$

C. Let $Y_{11}, \ldots, Y_{1n_1} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$Y_{21} \ldots Y_{2,n_2} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$

$\bar{Y}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} Y_i \qquad \bar{Y}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} Y_i$

$\bar{Y}_1 - \bar{Y}_2$ estimates $\mu_1 - \mu_2$

$E[\bar{Y}_1 - \bar{Y}_2] = \mu_1 - \mu_2 . \qquad V(\bar{Y}_1 - \bar{Y}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

D. Let Y_1 be the number of occurrences of the event 1 in n_1 independent trials, with probability p_1

Let Y_2 be the number of occurrences of the event 2 in n_2 independent trials, with probability of occurrence p_2

To estimate $p_1 - p_2$, we use $\frac{Y_1}{n_1} - \frac{Y_2}{n_2}$

$$E\left[\frac{Y_1}{n_1} - \frac{Y_2}{n_2}\right] = p_1 - p_2 \qquad V\left(\frac{Y_1}{n_1} - \frac{Y_2}{n_2}\right) = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$