8-17 (824x883x24b jpeg)

Confidence intervals

Interval estimators are commonly called confidence intervals.
Given the sample, we obtain $[\hat{\theta}_L, \hat{\theta}_U]$

The probability that a confidence interval will enclose θ is called the confidence coefficient

$$P(\hat{\theta}_L \le \theta \le \hat{\theta}_U) = 1-\alpha$$

We want to choose confidence intervals:
- large enough so that the confidence coefficient is $1-\alpha$ (we want to make sure that the confidence interval contains the parameter)

* narrow enough to make accurate predictions

A lower one-sided confidence interval of θ is $[\hat{\theta}_L, \infty)$

An upper one-sided confidence interval of θ is $(-\infty, \hat{\theta}_U]$

A method for finding confidence intervals is the pivotal method

A pivotal quantity is a function of the data and the target parameter whose distribution does not depend on θ.

8-18 (811x896x24b jpeg)

Confidence interval for the normal r.v.'s

Given $0 < \alpha < 1$, z_α is the solution of $P(Z \geq z_\alpha) = \alpha$

We will use that $P(-z_{\frac{\alpha}{2}} \leq Z \leq z_{\frac{\alpha}{2}}) = 1-\alpha$

Given a random sample $Y_1 \ldots Y_n$ from a normal distribution we would like to estimate μ and σ^2

Confidence intervals of μ, assuming σ^2 is known

(1) $[\bar{Y} - \frac{\sigma z_\alpha}{\sqrt{n}}, \infty)$ is a $1-\alpha$ lower-one side confidence interval for μ

(2) $(-\infty, \bar{Y} + \frac{\sigma z_\alpha}{\sqrt{n}}]$ is a $1-\alpha$ upper-one side confidence interval for μ

(3) $[\bar{Y} - \frac{\sigma z_{\alpha/2}}{\sqrt{n}}, \bar{Y} + \frac{\sigma z_{\alpha/2}}{\sqrt{n}}]$ is $1-\alpha$ confidence interval for μ

(1) $P(\bar{Y} - \frac{\sigma z_\alpha}{\sqrt{n}} \leq \mu) = P(\sqrt{n}\frac{(\bar{Y}-\mu)}{\sigma} \leq z_\alpha) = 1-\alpha$

(2) $P(\bar{Y} + \frac{\sigma z_\alpha}{\sqrt{n}} \geq \mu) = P(\sqrt{n}\frac{(\bar{Y}-\mu)}{\sigma} \geq -z_\alpha) = 1-\alpha$

(3) $P(\bar{Y} - \frac{\sigma z_{\alpha/2}}{\sqrt{n}} \leq \mu \leq \bar{Y} + \frac{\sigma z_{\alpha/2}}{\sqrt{n}}) = P(-z_{\frac{\alpha}{2}} \leq \frac{\sqrt{n}(\bar{Y}-\mu)}{\sigma} \leq z_{\frac{\alpha}{2}}) = 1-\alpha$

8-19 (803x601x24b jpeg)

Exercise

Let Y equal the amount of orange juice (in grams per day) consumed by an American. Suppose that it is known that the standard deviation of Y is $\sigma = 96$. To estimate the mean μ of Y an orange growers' association took a random sample of $n = 576$ Americans and found that they consumed, on the average $\bar{y} = 133$ grams of orange juice per day. Find a 90% confidence interval for μ

$\bar{y} = 133$, $\sigma = 96$, $n = 576$ $z_{\frac{\alpha}{2}} = z_{0.05} = 1.645$

$\bar{y} \pm \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}$,

A 90% confidence interval for μ is

$133 \pm (1.645)\frac{(96)}{\sqrt{576}} = 133 \pm 6.58 = [126.42, 139.58$

8-20 (778x883x24b jpeg)

Exercise

A random sample of size 8 from $N(\mu, \sigma^2 = 72)$, yielded $\bar{y} = 85$,

Find the following confidence intervals for μ

(a) 99% (b) 95% (c) 90% (d) 80%

(a) $\bar{y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 85 \pm (2.575) \frac{\sqrt{72}}{\sqrt{8}} = 85 \pm 7.725$

$= [77.275, 92.725]$

(b) $\bar{y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 85 \pm (1.96) \frac{\sqrt{72}}{\sqrt{8}} = 85 \pm 5.88$

$[79.12, 90.88]$

(c) $\bar{y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 85 \pm (1.645) \frac{\sqrt{72}}{\sqrt{8}} = 85 \pm 4.935$

$[80.065, 89.935]$

(d) $\bar{y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 85 \pm (1.282) \frac{\sqrt{72}}{\sqrt{8}} = 85 \pm 3.846$

$[8 6154, 88.846]$