Confidence intervals of μ, unknown σ^2

(1) $\left[\bar{Y} - \frac{S}{\sqrt{n}} t_{n-1}(\alpha), \infty\right)$ is a $1-\alpha$ lower one side confidence interval for μ

(2) $\left(-\infty, \bar{Y} + \frac{S}{\sqrt{n}} t_{n-1}(\alpha)\right]$ is a $1-\alpha$ upper one side confidence interval for μ

(3) $\left[\bar{Y} - \frac{S}{\sqrt{n}} t_{n-1}(\alpha), \bar{Y} + \frac{S}{\sqrt{n}} t_{n-1}(\alpha)\right]$ is a $1-\alpha$ confidence interval for μ

Confidence intervals of σ^2,

(1) $\left[\frac{(n-1)S^2}{\chi^2_{\alpha}(n-1)}, \infty\right)$ is a $1-\alpha$ lower confidence interval for σ^2

(2) $\left(-\infty, \frac{(n-1)S^2}{\chi^2_{1-\alpha}(n-1)}\right]$ is a $1-\alpha$ upper confidence interval for σ^2

(3) $\left[\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S^2}{1-\chi^2_{\frac{\alpha}{2}}(n-1)}\right]$ is a $1-\alpha$ confidence interval for σ^2

8.70. Organic chemists often purify organic compounds by a method known as fractional crystalization. An experimenter wanted to prepare and purify 4.85 grams of aniline. Ten 4.85 g of specimens of aniline were prepared and purified to produce acetanilide. The following dry yields were obtained

3.85 3.88 3.90 3.62 3.72 3.80 3.85 3.36 4.01 3.82

Construct a 95% confidence interval for the mean number of grams of acetanilide that can be recovered from 4.85 g of aniline

$\sum y_i = 37.81 \qquad \sum y_i^2 = 143.2543$

$\bar{y} = \frac{37.81}{10} = 3.781$

$\sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{1}{n} (\sum y_i)^2 = 143.2543 - \frac{1}{10} (37.81)^2 = 0.29469$

$s^2 = \frac{0.29469}{9} = 0.0327$

The 95% confidence interval is

$\bar{y} \pm t_{\frac{\alpha}{2}}(n-1) \frac{s}{\sqrt{n}} = 3.781 \pm 2.262 \sqrt{\frac{0.0327}{10}} = 3.781 \pm 0.1293$

$= (3.6517, 3.9103)$

8.79. Recently, the EPA set a maximum noise level for heavy trucks at 83 decibels. A random sample of six heavy trucks produced the following noise levels (in decibels

85.4 86.8 86.1 85.3 84.8 86.

Use these data to construct a 90% confidence interval for σ^2, the variance of the truck noise emission readings,

$\sum y_i = 514.4$ $\qquad \sum y_i^2 = 44103.74$

$\sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{1}{n}(\sum y_i)^2 = 44103.74 - \frac{1}{6}(514.4)^2 = 2.5133$

$\bar{y} = 85.7333$ $\qquad s^2 = \frac{2.5133}{5} = 0.5026$

$\chi^2_{0.95} = 1.145476$ $\qquad \chi^2_{0.10} = 11.0705$

$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}$ is $\frac{5 \cdot (0.5026)}{11.0750} \le \sigma^2 \le \frac{5(0.5026)}{1.145476}$

$0.2269 \le \sigma^2 \le 2.1938$

8.96. A chemical process has produced, on the average, 800 tons of chemical per day. The daily yields for the past week are 785, 805, 790 and 802 tons. Estimate the mean daily yield, with confidence coefficient 0.90 from the data. What assumptions did you make?

$\sum Y_i = 785 + 805 + 790 + 802 = 3182$

$\bar{Y} = \frac{\sum Y_i}{4} = 795.5$ is the estimate of the mean daily yield.

$\sum Y_i^2 = 785^2 + 805^2 + 790^2 + 802^2 = 2531554$

$\sum (Y_i - \bar{Y})^2 = \sum Y_i^2 - n(\bar{Y})^2 = 2531554 - 4(795.5)^2 = 273$

$s^2 = \frac{1}{3} \sum (Y_i - \bar{Y})^2 = \frac{273}{3} = 91$

The t 90% confidence interval for μ is

$\bar{Y} \pm t_{0.05} \frac{s}{\sqrt{n}} = 795.5 \pm 2353 \sqrt{\frac{91}{3}} = 795.5 \pm 12.96$

$(782.54, 808.46$

8.97. Refer to Exercise 8.96. Find a 90% confidence interval for σ^2, the variance of daily yields.

$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(3)$ $\quad P(0.351846 \le \chi^2(3) \le 7.81473) = 0.90$

$= P(0.351846 \le \frac{3 \cdot 91}{\sigma^2} \le 7.81473$

The confidence interval is $\left(\frac{3 \cdot 91}{7.81473}, \frac{3 \cdot 91}{0.351846}\right)$

$(34.93, 775.9076)$

8.67 Find a 95% confidence interval for the mean carapace length of T. orientalis lobsters from the data

carapace length m.m	78	66	65	63	60	60	58	56	52	50

$$\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n} y_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^2$$

$$\sum_{i=1}^{n}(y_i-\bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$$

$\sum y_i = 608 \qquad \bar{y} = 60.8$

$\sum y_i^2 = 37538$

$$\sum (y_i-\bar{y})^2 = 37538 - \frac{1}{10}(608)^2 = 571.6$$

$$s^2 = \frac{1}{n-1}\sum (y_i-\bar{y})^2 = \frac{1}{9}(571.6) = 63.5111$$

The 95% confidence interval is

$$\bar{y} \pm t_{\frac{\alpha}{2}}(n-1)\frac{s}{\sqrt{n}} = 60.8 \pm 2.262\sqrt{\frac{63.5111}{10}} = 60.8 \pm 5.7005$$

$= (55.0994, 66.5005)$