Asymptotic confidence intervals for the mean

Let $Y_1 \ldots Y_n$ be iid r.v's from a distribution with finite second moment. Let $\mu = E[Y]$ and let $\sigma^2 = Var(Y)$

(1) If σ^2 is known,

$$\bar{Y} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$ is an asymptotic $1-\alpha$ confidence interval for μ

(2) If σ^2 is unknown

$$\bar{y} \pm z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$ is an asymptotic $1-\alpha$ confidence interval for μ

where $S^2 = \frac{1}{n-1} \sum_{j=1}^{n} (Y_j - \bar{Y})^2$

Two samples

Let $Y_{11}, \ldots Y_{1,n_1}$ be a random sample from a distribution with finite seconde moment. Let $\mu_1 = E[Y_{1j}]$ and let $\sigma_1^2 = Var(Y_{1j})$

Let $Y_{21} \ldots Y_{2,n_2}$ be a random sample from a distribution with finite second moment. Let $\mu_2 = E[Y_{2j}]$ and let $\sigma_2^2 = Var(Y_{2j})$

Case 1 If σ_1^2 and σ_2^2 are known

$$\bar{y}_1 - \bar{y}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$ is a $1-\alpha$ confidence interval for $\mu_1 - \mu_2$

Case 2 If σ_1^2 and σ_2^2 are unknown

$$\bar{y}_1 - \bar{y}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$ is a $1-\alpha$ confidence interval for $\mu_1 - \mu_2$

$$S_1^2 = \frac{1}{n_1 - 1} \sum_{j=1}^{n_1} (Y_{1j} - \bar{Y}_1)^2 \qquad S_2^2 = \frac{1}{n_2 - 1} \sum_{j=1}^{n_2} (Y_{2j} - \bar{Y}_2)^2$$

8.42. The administrators for a hospital wished to estimate the average number of days required for in-patient treatment of patients between the ages of 25 and 34. A random sample of 500 hospital patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn

$\bar{Y} = 5.4$, $s = 3.1$, $n = 500$

$\sigma_{\bar{y}} = \frac{3.1}{\sqrt{500}} = 0.1386$

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

The 95% confidence interval is $\bar{Y} \pm \frac{s}{\sqrt{n}} 1.96$

$= 5.4 \pm (1.96)(0.1386) = 5.4 \pm 0.27 = (5.13, 5.67)$

8.45. A small amount of the trace element selenium, from 50 to 200 micrograms per day is considered essential to good health.

Suppose that random samples of $n_1 = n_2 = 30$ adults were selected from two regions of the US. The mean and standard deviation of the selenium daily intakes for the 30 adults from Region 1 were

$\bar{y}_1 = 167.1\ \mu g$ and $s_1 = 24.3\ \mu g$, respectively.

The corresponding statistics for the 30 adults from Region 2 were

$\bar{y}_2 = 140.9\ \mu g$ and $s_2 = 17.6\ \mu g$.

Find a 95 % confidence interval for the difference in the mean selenium intake for the two regions.

The estimator is $\bar{Y}_1 - \bar{Y}_2 = 167.1 - 140.9 = 26.2$

$V(\bar{Y}_1 - \bar{Y}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$, which is estimated by

$$\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} = \frac{24.3^2}{30} + \frac{17.6^2}{30} = 30.00$$

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

The 95% confidence interval for $\mu_1 - \mu_2$ is

$$26.2 \pm 1.96\sqrt{30.00} = 26.2 \pm 10.73 = (15.47, 36.93$$

8.54 Alice Chang tested 559 high school students using a leisure-interest checklist surveys. The means and standard deviations for each of the seven LIC factor scales are given for $n_1 = 252$ male students and $n_2 = 307$ female students:

	Males		Females	
factor	Mean	S	Mean	S
Cultural	11.48	5.69	13.21	5.31
Social fun	22.05	5.12	25.96	5.07

(a) Give a 95% confidence interval for the difference in the means for male and female students for the cultural activity scale.

(b) Find a 90% confidence interval for the difference in the means for male and female students for the social factor scale activity

(a) $\bar{y}_1 - \bar{y}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 11.48 - 13.21 \pm 1.96 \sqrt{\frac{(5.69)^2}{252} + \frac{(5.31)^2}{307}}$

$= -1.37 \pm 0.92$ or $[-2.65, -0.81]$

(b) $\bar{y}_1 - \bar{y}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 22.05 - 25.96 \pm 1.645 \sqrt{\frac{(5.12)^2}{252} + \frac{(5.07)^2}{307}}$

$= -3.91 \pm 0.71$ or $[-4.62, -3.20]$