Example 9.15

Let $Y_1, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance σ^2. Find the MLE's of μ and σ^2.

$$L(\mu, \sigma^2) = \prod_{i=1}^{n} \frac{e^{-\frac{(y_i-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma} = \frac{e^{-\frac{\sum (y_i-\mu)^2}{2\sigma^2}}}{(\sqrt{2\pi})^n \sigma^n}$$

$$\ln L(\mu, \sigma^2) = -\sum_{i=1}^{n} \frac{(y_i-\mu)^2}{2\sigma^2} - \frac{n}{2} \ln \sigma^2 - n \ln(\sqrt{2\pi})^n$$

$$\frac{\partial}{\mu} \ln L(\mu, \sigma^2) = -\sum_{i=1}^{n} \frac{\mu - y_i}{\sigma^2} = -\left(\frac{n\mu - \sum y_i}{\sigma^2}\right) = 0 \qquad \hat{\mu} = \bar{Y}$$

$$\hat{\sigma} = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

$$\frac{\partial}{\partial \sigma^2} \ln L(\mu, \sigma^2) = \sum_{i=1}^{n} \frac{(y_i-\mu)^2}{2\sigma^4} - \frac{n}{2\sigma^2}$$

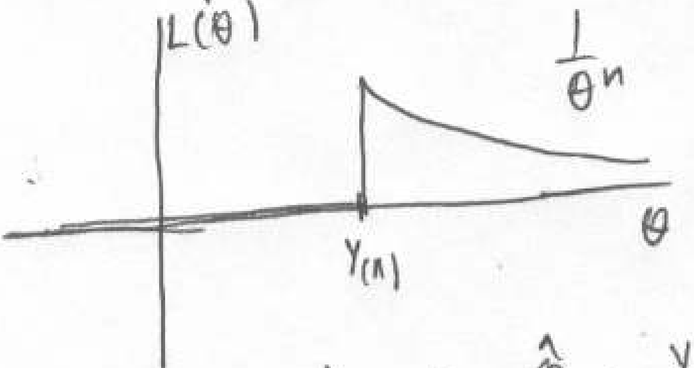
Example 9.16

Let $Y_1, \dots, Y_n$ be a random sample of observations from a uniform distribution with probability density function

$f(y_i \mid \theta) = \frac{1}{\theta}$, for $0 \le y_i \le \theta$, and $i = 1, \dots, n$.

$\boxed{\text{Find the MLE of } \theta}$

$$L(\theta) = \prod_{j=1}^{n} \frac{1}{\theta} I(0 < Y_j < \theta) = \frac{1}{\theta^n} I(0 < Y_{(1)}, Y_{(n)} < \theta)$$



The MLE of θ is $\hat{\theta}_n = Y_{(n)}$

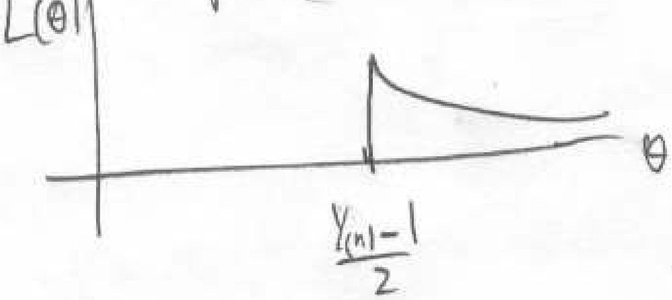
9.75 Suppose that $Y_1, \dots, Y_n$ constitute a random sample from a uniform distribution with probability density function

$$f(y|\theta) = \begin{cases} \dfrac{1}{2\theta+1} & \text{if } 0 \le y \le 2\theta+1 \\ 0 & \text{else} \end{cases}$$

(a) Obtain the maximum-likelihood estimator of θ

(b) Obtain the MLE for the variance of the underlying distribution

(a) $$L(\theta) = \prod_{j=1}^{n} \left(\frac{1}{2\theta+1} I(0 \le Y_{(j)} \le 2\theta+1) \right) = \frac{1}{(2\theta+1)^n} I(0 \le Y_{(1)}, Y_{(n)} \le 2\theta+1)$$



$$\hat{\theta} = \frac{Y_{(n)}-1}{2}$$

(b) $$\operatorname{Var}(Y) = \frac{(2\theta+1)^2}{12}$$

The MLE of $\frac{(2\theta+1)^2}{12}$ is $$\frac{\left(2\left(\frac{Y_{(n)}-1}{2}\right)+1\right)^2}{12} = \frac{Y_{(n)}^2}{12}$$

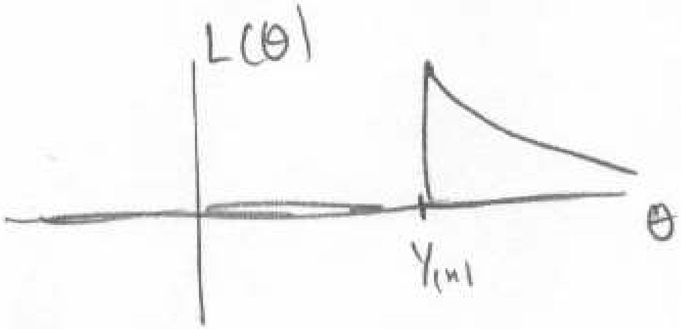
Example

Let $Y_1 \ldots Y_n$ denote a random sample from the density

$$f(y|\theta) = \begin{cases} \frac{2y}{\theta^2} & \text{if } 0 < y < \theta \\ 0 & \text{else} \end{cases}$$

Find the mle of θ

$$L(\theta) = \prod_{i=1}^{n} \frac{2y}{\theta^2} I(0 < Y_i \le \theta) = \frac{2^n \prod Y_i}{\theta^{2n}} I(0 < Y_{(1)}, Y_{(n)} \le \theta)$$



$$\hat{\theta} = Y_{(n)}$$

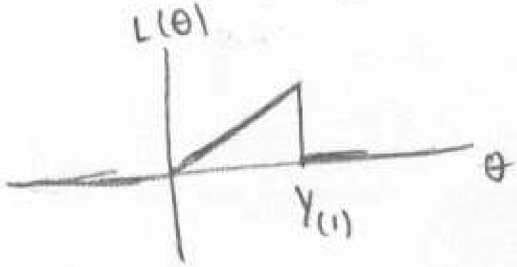
Example

Let $Y_1 \ldots Y_n$ denote a random sample from the density

$$f(y|\theta) = \begin{cases} \dfrac{\alpha\,\theta^{\alpha}}{y^{\alpha+1}} & \text{if } \theta < y \\ 0 & \text{else} \end{cases}$$

Find the mle of θ, where α is a known parameter and $\theta > 0$

$$L(\theta) = \prod_{i=1}^{n} \frac{\alpha\,\theta^{\alpha}}{Y_i^{\alpha+1}} I(\theta < Y) = \frac{\alpha^n\,\theta^{n\alpha}}{\prod Y_i^{\alpha+1}} I(\theta < Y_{(1)})$$



$$\hat{\theta} = Y_{(1)}$$