e1s-1 (706x803x24b jpeg)

1.

$$1 = \int_0^\infty c x^7 e^{-x/2} dx = c \, 7! \, 2^8 \qquad c = \frac{1}{7! \cdot 2^8}$$

$$E[X] = \int_0^\infty x \, c x^7 e^{-x/2} dx = c \; 8! \, 2^9 = \frac{8! \, 2^9}{7! \, 2^8} = 16$$

2. The range of U is (e^1, e^4)

$$U = e^{5-Y} = h(Y) \qquad Y = 5 - \ln u$$

$$f_U(u) = \frac{1}{3} \left| \frac{-1}{u} \right| = \frac{1}{3u} \quad \text{if} \quad e < u < e^4$$

3. $Y_1 = \frac{X_1}{X_2}$, $Y_2 = X_2$ $\Big\}$ $X_1 = Y_1 Y_2$, $X_2 = Y_2$

$$J = \det \begin{pmatrix} y_2 & y_1 \\ 0 & 1 \end{pmatrix} = y_2$$

$$f_{Y_1, Y_2}(y_1, y_2) = 8 \, y_1 y_2 y_2 y_2 \quad \text{if} \quad 0 < y_1 y_2 < y_2 < 1 \qquad \begin{matrix} 0 < y_1 < 1 \\ 0 < y_2 < 1 \end{matrix}$$

$$= 8 y_1 y_2^3$$

$$f_{Y_1, Y_2}(y_1, y_2) = \begin{cases} 8 \, y_1 y_2^3 & \text{if } 0 < y_1 < 1, \text{ and } 0 < y_2 < 1 \\ 0 & \text{else} \end{cases}$$

e1s-2 (714x993x24b jpeg)

4. $E[X_i] = 50 \quad Var(X) = 16$

$E[\sum_1^{25} X_i] = (25)(50) = 1250$

$Var(\sum_1^{25} X_i) = 25(16) = 400$

$P(\sum_{i=1}^{25} X_i \geq 1300) = P(N(0,1) \geq \frac{1300-1250}{\sqrt{40}})$

$= P(N(0,1) \geq 2.5) = 0.0062$

5. $S =$ no. of cars who turn right

$S \sim Bin(n=500, p=\frac{1}{3})$

$E[S] = np = 500\,\frac{1}{3} = 166.67$

$Var(S) = np(1-p) = 500 \cdot \frac{1}{3} \cdot \frac{2}{3} = 111.11$

$P(S \leq 150) = P(S \leq 150.5) = P(N(0,1) \leq \frac{150.5-166.67}{\sqrt{111.11}})$

$= P(N(0,1) \leq -1.534) = 0.0630$

6. $\mu = 10, \sigma = 0.1 \quad E[\bar{Y}] = 10 \quad Var(\bar{Y}) = \frac{\sigma^2}{n} = 0.001$

$P(\bar{Y} < 9.93) = P(N(0,1) \leq \frac{9.93-10}{\sqrt{0.001}}) = P(N(0,1) \leq -2.2136)$

$= 0.0136$

e1s-3 (714x820x24b jpeg)

7.

$E[X] = np = 500$ $\quad Var(X) = np(1-p) = 250$

$0.70 \leq P(500-a \leq X \leq 500+a) = 1 - 2P(X \geq 500+a)$

$P(X \geq 500+a) \leq \frac{1-0.70}{2} = 0.15$

$\shortparallel$

$P\left(N(0,1) \geq \frac{a-0.5}{\sqrt{250}}\right) = 0.15$

$\frac{a-0.5}{\sqrt{250}} = 1.035$

$a = 0.5 + (1.035)\sqrt{250} = 16.86$

$a = 17$

8. $\sum_{i=1}^{4} (X_i - \bar{X})^2 \sim \chi^2(3)$, $\sum_{j=1}^{10} (Y_j - \bar{Y})^2 \sim \chi^2(9)$

$\sum_{i=1}^{4} (X_i - \bar{X})^2 + \sum_{j=1}^{10} (Y_j - \bar{Y})^2 \sim \chi^2(12)$