Math 448. First Midterm. Friday, February 20, 2004.

Name: ....

Show all your work. No credit for lucky answers.

1. The lifetime in hours of an electronic part is a random variable having a probability density function given by $f(x) = cx^7 e^{-x/2}, \quad x > 0$. Compute c and the mean of the lifetime of such an electronic part.

2. Let Y be a random variable with a density function given by

$$f(y) = \begin{cases} \frac{1}{3} & \text{if } 1 \leq y \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

   Find the density of $U = e^{5-Y}$.

3. Let X_1 and X_2 have the joint pdf

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   Find the joint pdf of $Y_1 = \frac{X_1}{X_2}$ and $Y_2 = X_2$ and argue that of Y_1 and Y_2 are independent.

4. Twenty-five hear lamps are connected in a greenhouse of that when one lamp fails, another takes over immediately. Only one lamp is turned on at any time. The lamps operate independently, and each has a mean life of 50 hours and a standard deviation of 4 hours. If the greenhouse is not checked for 1300 hours after the lamp system is turned on, what is the probability that a lamp will be burning at the end of the 1300-hour period.

5. Vehicles entering an intersection from the east are equally likely to turn left, turn right, or proceed straight ahead. If 500 vehicles enter this intersection from the east, what is the approximate probability that 150 or fewer turn right?

6. The mean number of ounces of coffee a vending machine dispenses is 10. The standard deviation is 0.1. If a sample of 10 cups is selected, find the probability that the mean of the sample will be less than 9.93 ounces.

7. One thousand independent throws of a fair coin will be made. Let S be the number of heads in these thousand independent throws. Find the smallest value of a so that

$$P[500 - a \leq S \leq 500 + a] \geq 0.70.$$

8. Let $X_1, \ldots, X_4, Y_1, \ldots, Y_{10}$ be a random sample from a N(0,1) distribution. Let $\bar{X} = \frac{1}{4}\sum_{i=1}^{4} X_i$ and let $\bar{Y} = \frac{1}{j}\sum_{j=1}^{10} X_j$. Find the distribution of

$$\sum_{i=1}^{4}(X_i - \bar{X})^2 + \sum_{j=1}^{10}(Y_j - \bar{Y})^2.$$