Math 448. Final Exam. Tuesday, May 11, 2004. 11:00 am-01:00 pm at AAG007.

Name: ....

Show all your work. No credit for lucky answers.

1. Potential advertisers value television's well-known Nielsen ratings as a barometer of a TV show's popularity among viewers. The Nielsen rating of a certain TV program is an estimate of the proportion of viewers, expressed as a percentage, who tune their sets to the program on a given night. Suppose the Nielsen ratings are to be found for a premiere of a new hospital drama show using a simple random sample. If no information is known about the possible rating, what sample size should the researcher use for a 90% confidence interval in order to obtain a margin of error of 6%.

2. A national poll commissioned by the Center on Addiction and Substance Abuse at Columbia University found that 1340 out of 2000 adults interviewed believed that popular culture encourages drug use.

(a) Find a 98% confidence interval for the true proportion of adults with this belief at that time.

(b) Is there enough evidence at the 10% level to claim that more that 2/3 of adults believe that popular culture encourages drug use?

3. In a random sample of 16 cardiac patients who participated in an exercise program, the mean weight loss was 25 pounds. The sample estimate of the population variance was 13.04.
(a) Construct a 95% confidence interval for the mean.
(b) For the test whether the mean weight loss for the exercise program is more 20 pounds, estimate the p–value.

4. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{4y^3 e^{-\frac{y^4}{\theta}}}{\theta^2} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter.
(a) Find a sufficient statistic of θ.
(b) Find the MVUE of θ (find a function of the sufficient statistic which an unbiased estimator of θ).

5. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{4y^3}{\theta^4}, & \text{if } 0 \leq y \leq \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is an unknown parameter.
(a) Find the method of moments estimator $\hat{\theta}$ of θ.
(b) Is this estimator consistent for θ? (prove it!)
(c) Is this estimator unbiased for θ? (prove it!)
(d) Find the mean square error of $\hat{\theta}$.

6. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{\theta}{y^2}, & \text{if } y \geq \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is an unknown parameter.
(a) Find the maximum likelihood estimator of θ.
(b) Is this estimator consistent for θ? (prove it!)
(c) Is this estimator unbiased for θ? (prove it!)

7. The ACME light bulb company will shut its production process down if it has reason to believe that the mean lifetime of its bulbs is significantly different from 1000 hours. It is known that the lifetimes are normally distributed with standard deviation 300 hours. The company only wants to take a 5% risk of shutting down production when the process is in control. If a sample of 25 bulbs has an average lifetime of 1100 hours, will we shut down the process?

8. A growing body of evidence suggests that married persons live longer, with lower mortality for almost every major cause of death, in comparison with unmarried persons. Dr. Goodwin theorizes that marital status may affect cancer mortality by influencing survival following diagnosis of cancer. Dr. Goodwin examines data from a random sample of 26 cancer cases. Half of the cases involved married persons and half involved unmarried persons. The married persons had survived a mean of 8.40 years post diagnosis. The standard deviation for this set of data was 1.25. The unmarried persons survived a mean of 4.10 years post diagnosis. The standard deviation for this set of data was 1.31. Perform a statistical test to determine whether Dr. Goodwin should conclude that married persons differ from unmarried persons in length of survival following cancer diagnosis. Assume that the two sets of data are independent random samples from normal populations with equal variances.

   (a) What are the null and alternative hypothesis?

   (b) Estimate the p–value.

   (c) Would you reject the null hypothesis if the significance level is (a) $\alpha = 0.10$? (b) $\alpha = 0.05$? (c) $\alpha = 0.01$?

9. An experimenter has prepared a drug dosage level that she claims will suffering from insomnia. After examining the dosage, we feel that her claims the dosage are inflated. In an attempt to disprove her claim, we administer insomniacs, and we observe Y , the number for which the drug dose induces sleep. We wish to test the hypothesis $H_0 : p = 0.8$ versus the alternative $H_1 : p < 0.8$. Determine n so that the type I error is 0.01 and the type II error when $p = 0.6$ is 0.02.

10. Let $X_1, \ldots, X_n$ be a random sample from the density

$$f(y|\theta) = \begin{cases} \theta^2 y e^{-\theta y} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter.

(a) Find the maximum likelihood estimator of θ.

(b) Find the rejection region for the likelihood ratio test of $H_0 : \theta = \theta_0$ against $H_a : \theta \neq \theta_0$, where $\theta_0 > 0$.(we just to write the rejection region in the form $\{(y_1, \ldots, y_n) : T(y_1, \ldots, y_n) \leq k\}$, where $T(y_1, \ldots, y_n)$ is a statistic which you need to find).