h1-1 (752x765x24b jpeg)

Homework 1

1. The density of Z is $f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$

$E[Z^3] = \int_{-\infty}^{\infty} z^3 \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = 0$, because $z^3 e^{-z^2/2}$ is an odd function

$E[Z^4] = \int_{-\infty}^{\infty} z^4 \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz = \int_{-\infty}^{\infty} z^3 \frac{d(-e^{-z^2/2})}{\sqrt{2\pi}} = \left. \frac{z^3(-e^{-z^2/2})}{\sqrt{2\pi}} \right]_{-\infty}^{\infty}$

$\int_{-\infty}^{\infty} \frac{(-e^{-z^2/2})}{\sqrt{2\pi}} 3z^2 dz = \int_{-\infty}^{\infty} \frac{3}{\sqrt{2\pi}} z^2 e^{-z^2/2} dz = 3$

2. X = length of the time required to - - -

$\mu = 70$, $\sigma = 12$, t = time at which test should be terminated

$0.90 = P(X \le t) = P\left(N(0,1) \le \frac{t-70}{12}\right)$

$\frac{t-70}{12} = 1.28$ $\qquad t = 85.36$

h1-2 (748x921x24b jpeg)

3. $E[Y_1]=1$, $E[Y_2]=-3$, $E[Y_3]=4$

$Var(Y_1)=1$, $Var(Y_2)=4$, $Var(Y_3)=2$

(a) $E[2-3Y_1+4Y_2+5Y_3] = 2+(-3)(1)+(4)(-3)+(5)(4)$

$= 2-3-12+20 = 7$

$Var(2-3Y_1+4Y_2+5Y_3)$

$= 9\,Var(Y_1)+16\,Var(Y_2)+25\,Var(Y_3)$

$= 9\cdot(1)+16(4)+25(2) = 9+64+50 = 123$

b) $E[Y_2^2] = Var(Y_2)+(E[Y_2])^2 = 4+(-3)^2 = 13$

$E[Y_1^2]=2$

$E[2-3Y_1^2+4Y_1Y_2+5Y_2^2Y_3]$

$= 2-3(2)+4(1)(-3)+5(13)(4) = 2-6-12+260 = 244$

4. $E[\bar{Y}]=3$ $Var(\bar{Y}) = \frac{40}{10} = 4$

$0.95 = P(\bar{Y} \le b) = P(N(0,1) \le \frac{b-3}{2}) = P(N(0,1) \le 2.575)$

$\frac{b-3}{2} = 1.645$ $b = 6.29$

h1-3 (799x871x24b jpeg)

5. $E[X_i] = 3125$ $\quad Var(X_i) = 250^2 = 62500$

$E[\sum_{i=1}^{2025} X_i] = (2025)(3125) = 6328125$

$Var(\sum_{i=1}^{2025} X_i) = (2025)(62500) = 126562500 = ((45)(250))^2$

$0.90 = P(N(0,1) \le 1.282)$

$\shortparallel$

$P(X \le p_{0.90}) = P\left(N(0,1) \le \frac{p_{0.90} - 6328125}{\sqrt{126562500}}\right)$

$p_{0.90} = 6328125 + (1.282)\sqrt{126562500}$

$= 6342547.5$

6. $\bar{Y} \sim N(2.2, \frac{\sigma^2}{n} = \frac{0.3^2}{100} = 0.0009$

$P(\bar{Y} > 3.1) = P\left(N(0,1) > \frac{3.1 - 2.2}{\sqrt{0.0009}}\right) = P\left(N(0,1) \geq \frac{0.9}{\frac{0.3}{10}}\right)$

$= P(N(0,1) \geq 30) = 0$

h1-4 (748x1081x24b jpeg)

7. $X_1 \ldots X_{15} \sim N(70, \sigma = 25)$

$Y_1 \ldots Y_{40} \sim N(70, \sigma = 25)$

$\bar{x} \sim N(70, \frac{\sigma^2}{n} = \frac{25^2}{15} = 41.67)$

$\bar{y} \sim N(70, \frac{\sigma^2}{n} = \frac{25^2}{40} = 15.625)$

$P(\bar{y} - \bar{x} > 4) = P(N(0,1) \geq \frac{4}{\sqrt{57.295}}) = P(N(0,1) \geq 0.528)$

$\bar{y} - \bar{x} \sim N(0, 41.67 + 15.625 = 57.295$

$= 0.2981$

8. $1 = \int_0^\infty c x^9 e^{-x} = c\, 9! \qquad c = \frac{1}{9!}$

$X \sim Gamma(10, 1) \qquad E[X] = 10 \quad Var(X) = 10$

$\int_0^\infty x^{\alpha-1} e^{-\frac{x}{\theta}} dx = \Gamma(\alpha)\, \theta^\alpha$

9. $f(x) = 1.4\, e^{-2x} + 0.9\, e^{-3x}$

$E[X] = \int_0^\infty x\,(1.4\, e^{-2x} + 0.9\, e^{-3x})\,dx = 1.4\left(\frac{1}{2}\right)^2 + 0.9\left(\frac{1}{3}\right)^2$

$= 0.35 + 0.1 = 0.45$

$E[X^2] = \int_0^\infty x^2 (1.4\, e^{-2x} + 0.9\, e^{-3x})\,dx = (1.4)\,2\cdot\left(\frac{1}{2}\right)^3 + (0.9)\cdot 2\cdot\left(\frac{1}{3}\right)^3$

$= 0.35 + 0.0667 = 0.4167 = 5/12$

$Var(X) = 0.4167 - (0.45)^2 = 0.2164$

h1-5 (680x345x24b jpeg)

10. $\int_0^\infty x^{\tau-1} e^{-\theta x^\tau} dx = \int_0^\infty \frac{1}{\tau\theta} e^{-y} dy = \frac{1}{\tau\theta}$

$\theta x^\tau = y \quad x =$

$\theta \tau x^{\tau-1} dx = dy$

$$\int_0^\infty x^{\alpha\tau+\tau-1} e^{-\theta x^\tau} dx = \int_0^\infty \tau \left(\frac{y}{\theta}\right)^\alpha e^{-y} \frac{dy}{\theta} = \frac{\Gamma(\alpha+1)}{\tau\theta^{\alpha+1}}$$