Math–448. 1st Homework. Due Friday, February 6, 2004.

1. Let Z be a standard normal random variables. Find $E[Z^3]$ and $E[Z^4]$.

2. The length of the time required to complete a college achievement test is found to be normally distributed with mean 70 minutes and standard deviation 12 minutes. When should the test be terminated if we wish to allow sufficient time of 90 % of the students to complete the test?

3. Let Y_1, Y_2, Y_3 be independent random variables. Suppose that $E[Y_1] = 1$, $E[Y_2] = -3$, $E[Y_3] = 4$, $\mathrm{Var}(Y_1) = 1$, $\mathrm{Var}(Y_2) = 4$ and $\mathrm{Var}(Y_3) = 2$. Find:

   (a) The mean and the variance of $2 - 3Y_1 + 4Y_2 + 5Y_3$.

   (b) $E[2 - 3Y_1^2 + 4Y_1Y_2 + 5Y_2^2Y_3]$.

4. Let $Y_1, \ldots, Y_{10}$ be independent identically distributed random variables with a normal distribution with mean 3 and variance 40. Find b so that $P\{\bar{Y} \le b\} = 0.95$.

5. A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250. Calculate the approximate 90th percentile for the distribution of the total contributions received. Hint: The 90th percentile $p_{0.90}$ of a r.v. is the solution of the equation $P[X \le p_{0.90}] = 0.90$.

6. A tobacco company claims that the amount of nicotine in its cigarettes is a random variable with mean 2.2 mg. and standard deviation .3 mg. However, the sample mean nicotine content of 100 randomly cigarettes was 3.1 mg. What is the approximate probability that the sample mean would have been as high or higher than 3.1 if the company's claims were true?

7. Students scores on exams given by a certain instructor have normal distribution with mean 70 and standard deviation 25. This instructor is about to give two exams, one for a class of size 15 and the other to a class of size 40. Approximate the probability that the average test score in the larger class exceeds that of the other class by over 4 points.

8. The lifetime in hours of an electronic part is a random variable having a probability density function given by $f(x) = cx^9e^{-x}, \quad x > 0$. Compute c and the mean and the variance of the lifetime of such an electronic part.

9. If X is a random variable with density function

$$f(x) = \begin{cases} 1.4e^{-2x} + .9e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

   Find the mean and the variance of X.

10. Find $\int_0^\infty x^{\tau-1}e^{-\theta x^\tau}$ and $\int_0^\infty x^{\alpha\tau+\tau-1}e^{-\theta x^\tau}$.