Math–448. 2nd Homework. Due Friday, February 20, 2004.

1. Let F be a random variable with ν_1 and ν_2 numerator and denominator degress of freedom respectively. Show that $E[F] = \frac{\nu_2}{\nu_2 - 2}$, if $\nu_2 > 2$.

2. Let Y be a random variable with a density function given by

$$f(y) = \begin{cases} \frac{3}{2}y^2 & \text{if } -1 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the density of $U = 5 - 3Y$.
   (b) Find the density of $U = 5 - e^Y$.
   (c) Find the density of $U = 3Y^2 - 4$.

3. The exponential density function is given by

$$f(y) = \begin{cases} \frac{1}{\beta}e^{-\frac{y}{\beta}} & \text{if } 0 \le y \\ 0 & \text{elsewhere} \end{cases}$$

   The Weibull density function is given by

$$f(y) = \begin{cases} \frac{1}{\alpha}my^{m-1}e^{-\frac{y^m}{\alpha}} & \text{if } 0 \le y \\ 0 & \text{elsewhere} \end{cases}$$

   Let Y have an exponential distribution with mean β. Prove that $U = Y^{\frac{1}{m}}$ has a Weibull distribution density with parameters m and $\alpha = \beta$.

4. The length pf life of two different types of components operating in a system has probability density function

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} \frac{1}{8}y_1e^{-\frac{y_1+y_2}{2}} & \text{if } 0 < y_1, 0 \le y_2 \\ 0 & \text{elsewhere} \end{cases}$$

   Let $U_1 = Y_1$ and let $U_2 = \frac{Y_2}{Y_1}$. Find the joint density of (U_1, U_2). Find the marginal probability density function of U_2.

5. Suppose you have \$100,000 to invest in stocks. If you invest \$1,000 in any particular stock, your profit will be \$200, \$100, \$0, or -\$100, with probability 1/4 each. There are 100 different stocks you can choose from, and they all behave independently of each other. Find the probability that your profit will be \$8000 or more (a) if you invest \$100,000 in one stock, (b) if you invest \$1,000 in each of 100 stocks.

6. One thousand independent rolls of a fair die are made. Let X be the number of times that the number 6 appears. Using the central limit theorem estimate the probability that $150 \le X < 200$.

7. Let $Y_1, \dots, Y_{200}$ independent identically distributed random variables with exponential distribution with mean 0.50. Find (approximately)

$$P[Y_1 + \cdots + Y_{200} \geq 110].$$

8. A university that is better known for its sports program than its academic strength claims that 80 % of its athletes get degrees. An investigation examines the fate of 50 athletes who entered the program over a period of several years that ends 5 years ago. Of these players, 30 graduated and 20 are no longer in school. Assuming that the university's claim is true, find the probability that 30 or less athletes graduated, out of 50 athletes randomly selected.

9. A new elevator in a large hotel is designed to carry about 30 people, with a total weight of up to 5000 lbs. More than 5000 lbs. overloads the elevator. The average weight of the guests at this hotel is 150 lbs., with a standard deviation of 55 lbs. Suppose 30 of the hotel's guests get into the elevator. Assuming the weights of these guests are independent random variables, what is the chance of overloading the elevator?

10. Let $\bar{x}$ be the mean of a random sample of size n from a normal distribution with mean 100 and variance $\sigma^2 = 4$. Find the smallest n so that $\Pr\{99.9 \leq \bar{x} \leq 100.1\} \geq .95$.