Math–448. 3rd Homework. Due Wednesday, March 10, 2004.

1. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{3y^2}{\theta^3} & \text{if } 0 < y < \theta, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter. Find constants a and b such that $\hat{\theta}_1 = a\bar{Y}$ and $\hat{\theta}_2 = bY_{(n)}$ are both unbiased estimators of θ. Find the mean square error of $\hat{\theta}_1$ and $\hat{\theta}_2$? Which of the estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ has smallest mean square error? Which estimator is preferred?

2. Given a random sample of size n from the density

$$f(y|\theta) = \begin{cases} \frac{my^{m-1}e^{-\frac{y^m}{\theta}}}{\theta} & \text{if } 0 < y, \\ 0 & \text{else.} \end{cases}$$

where $\theta > 0$ is unknown parameter and $m > 0$ is known. Show that $\hat{\theta} = \frac{1}{n}\sum_{j=1}^n Y_j^m$ is an unbiased estimator of θ. Find the mean square error of $\hat{\theta}$.

3. Let $Y_1, \dots, Y_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance $\sigma^2 > 0$. Find a such that $as = a\sqrt{\frac{1}{n-1}\sum_{j=1}^n (Y_j - \bar{Y})^2}$ is an unbiased estimator of σ.

4. A farm grows grapes for jelly. The following data are measurements fo sugar in the grapes of a sample take from each of 30 truckloads:

16.0	15.2	12.0	16.9	14.4	16.3	15.6	12.9	15.3	15.1
15.8	15.5	12.5	14.5	14.9	15.1	16.0	12.5	14.3	15.4
15.4	13.0	12.6	14.9	15.1	15.3	12.4	17.2	14.7	14.8

Assume that there are observations of a $N(\mu, \sigma^2)$. Find point estimates for μ and σ^2. Find an approximate 90 % confidence interval for μ. Find an approximate 95 % confidence interval for σ^2.

5. A hospital administrator wishes to estimate the mean number of days that infants spend in ICUs.
(a) How many records should she examine to have 99% confidence that the estimate is not more the 0.5 days from the mean? Previous work suggest that the standard deviation is 1.6.
(b) How many records should she examine if she wants to lower the confidence interval to 95%?

6. In a random sample of 250 viewers in a large city, 190 had seen a certain controversial program. Construct a 95 % confidence interval of the true proportion of people that saw that program.

7. If 132 of 200 male voters and 90 of 159 female voters favor certain candidate running for governor of Illinois. Find a 99 % confidence interval for the difference between the actual proportion of male and female who favor the candidate.

8. A market research is interested in determining the proportion of households that are watching a particular sporting event. To accomplish this task, they plan on using a telephone poll of randomly chosen households. How large a sample is needed if they want to be 90 % percent certain that their estimate is correct to within $\pm .02$?

9. A study has been made to compare the nicotine contents of two brands of cigarettes. Ten cigarettes of Brand A had an average nicotine content of 3.1 milligrams with a standard deviation of 0.5 milligram, while eight cigarettes of Brand B had an average nicotine content of 2.7 milligram with a standard deviation of 0.7 milligrams. Assuming that the two sets of data are independent random samples from normal populations with equal variances, construct a 95 % confidence interval of the difference between the mean nicotine contents of the two brands of cigarettes. (Use the t distribution).

10. Let X and let Y equal the concentration in parts per billion of chromium in the blood for healthy persons and for persons with a suspected disease, respectively. Assume that the distribution of X and Y are $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$ respectively. Using $n = 9$ observations of X

$$15\ 23\ 12\ 18\ 9\ 28\ 22\ 11\ 10$$

and $m = 10$ observations of Y

$$25\ 20\ 35\ 15\ 40\ 16\ 10\ 22\ 18\ 32$$

(a) Give a point estimate of σ_X^2/σ_Y^2.
(b) Find a 95 % confidence interval for σ_X^2/σ_Y^2.